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On A Production Inventory Model with Time Dependent Demand Rate

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ABSTRACT
This paper deals with the development of an inventory model in which the production rate is flexible. The model is assumed over a finite planning horizon taking the production rate as a decision variable. Demand rate is stock-dependent and the stock undergoes decay or deterioration. It is assumed that the demand rate remains stock-dependent for an initial period after which a uniform demand rate follows as the stock comes down to a certain level. The mathematical expression for the average profit function and other parameter has been obtained.

Keywords *– Inventory, Demand, Production*

I. INTRODUCTION
Inventory is the resource of any kind a businessman would like to have to promote smooth and efficient management of this business. It is the stock of goods, commodities and other economic resource that are stored or reserved in order to ensure smooth and efficient running of business affairs. Raafat[1] and Goyal and Giri[2] presented a complete survey of the published inventory literature for deteriorating inventory model . At first, Wagner and Whitin[3] presented an inventory model for goods deteriorating at the end of storage period. Firstly, Ghare and Scharder[4] pointed out the effect of decay in inventory analysis.This model was extended by Covert and Philip[5] by considering Weibull deterioration.Dave and Patel[6], Dave[7], Bahari-Kashani[8] and Roy Choudhuri and Choudhuri [9] discussed such model with variable demand and deterioration rate is constant. Mandal et al. [10] and Singh S.R.et al.[11] considered two different types of inventory based on price dependent demand.

II. ASSUMPTIONS
1. The inventory system involves only one item and is a self-production system.
2. Lead time is zero.
3. No shortages are permitted.
4. The time-horizon is infinite.
5. The production cost per unit item is a function of the production rate.
6. The production rate is considered to be a decision variable.

III. NOTATIONS
$p = a.e^{bt}$ is the exponentially increasing production rate with respect to time 't'. Here a and b are positive constants and $a \geq b$.

$I(t)$ - On –hand inventory at time $'t' \geq 0$.

$R(I)$ - Demand-rate function varying with $I(t)$.

S_0- The stock-level, beyond which the demand rate becomes constant.

$\theta -$Constant deterioration rate of the On-hand inventory, $0 < \theta < 1$.

$C_h -$Holding cost per unit per unit time.

$C_s -$Setup cost per production run.

$\eta(P) -$The priduction cost per unit item.

$S_p -$Salvage cost per unit item.

$T -$The duration of the production cycle.

$\nabla -$Gradient operator.

IV. FORMULATION OF THE MODEL
We consider a self-manufacturing system in which the items are manufactured in a machine and the market demand is filled by these manufactured items. The demand rate is dependent on the On-hand inventory down to a level S_0, beyond which it is assumed to be a constant, i.e.,

$$R(I) = D + \gamma I(t), I > S_0$$
$$= D + S_0, 0 \leq I \leq S_0,$$

Here D and γ are non-negative constants and $D < P$.

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The production cost per unit is $\eta(P) = r + \frac{g}{P} + \alpha P$

Where r, g, and α are all positive constants. This cost is based on the following factors:

1. The material cost r per unit item is fixed.
2. As the production rate increases, some costs like labour and energy costs are equally distributed over a large number of units. Hence the production cost per unit (g/P) decreases as the production rate (P) increases.
3. The third term (αP), associated with tool/die costs, and is proportional to the production rate.

V. MATHEMATICAL FORMULATION AND ANALYSIS OF THE MODEL

The production cycle begins with zero. Production starts at $t = 0$, and the stock reaches a level S_0 at time $t = t_1$ after meeting demands. The demand rate in the interval $(0, t_1)$ is $(D + \gamma S_0)$. in the interval (t_1, t_2), production continues uninterruptedly and the demand rate depends on the instantaneous stock level. Production is stopped at time $t = t_2$. The demand rate continues to depend on the instantaneous inventory level until $t = t_3$ when the stock falls down to the level S_0 again. The inventory falls to the zero level at the end $t = T$ of the production cycle. This cycle of production is repeated over and aver again. Therefore, the governing equations of this model are given by

$$\frac{dI(t)}{dt} + \theta I(t) = a.e^{bt} - (D + \gamma S_0), 0 \le t \le t_1, I \le S_0, b = 0$$

With $I(0) = 0$ and $I(t_1) = S_0$ ………….. (1)

$$\frac{dI(t)}{dt} + (\theta + \gamma) I(t) = a.e^{bt} - D, t_1 \le t \le t_2, I > S_0, b = 0$$

. ……..……. (2)

$$\frac{dI(t)}{dt} + (\theta + \gamma) I(t) = -D, t_2 \le t \le t_3, I > S_0, \text{ With}$$

$I(t_3) = S_0$ …………..... (3)

$$\frac{dI(t)}{dt} + \theta I(t) = -(D + \gamma S_0), t_3 \le t \le T, I \le S_0,$$

With $I(T) = 0$ …...………. (4)

From (1)

$$I(t) = \left(\frac{a - D - \gamma S_0}{\theta}\right)(1 - e^{-\theta t}) \quad \text{…………... (5)}$$

Now applying the boundary condition $I(t_1) = S_0$

$$t_1 = -\frac{1}{\theta}\log\left(1 - \frac{S_0 \theta}{a - D - \lambda S_0}\right) \quad \text{…………..... (6)}$$

From (2)

$$I(t) = \left(S_0 - \frac{a - D}{\theta + \gamma}\right)e^{(\theta+\gamma)(t_1 - t)} + \left(\frac{a - D}{\theta + \gamma}\right)$$

……………... (7)

Therefore putting $t = t_2$, we get

$$I(t_2) = \left(S_0 - \frac{a - D}{\theta + \gamma}\right)e^{(\theta+\gamma)(t_1 - t_2)} + \left(\frac{a - D}{\theta + \gamma}\right)$$

…………….. (8)

From (3)

$$I(t) = \left[I(t_2) + \frac{D}{\theta + \gamma}\right]e^{(\theta+\gamma)(t_2 - t)} - \frac{D}{\theta + \gamma}$$

……………… (9)

Now putting $I(t) = S_0, t = t_3$, we get

$$t_3 = t_2 - \frac{1}{\theta + \gamma}\log\left[\frac{S_0 + \frac{D}{(\theta + \gamma)}}{I(t_2) + \frac{D}{(\theta + \gamma)}}\right] \quad \text{…..... (10)}$$

From (4)

$$I(t) = \left[S_0 + \frac{(D + \gamma S_0)}{\theta}\right]e^{\theta(t_3 - t)} - \left(\frac{D + \gamma S_0}{\theta}\right)$$

……………….. (11)

Now $I(t) = 0$, so putting $t = T$, we get

$$T = t_3 - \frac{1}{\theta}\log\left[\frac{D + \gamma S_0}{D + S_0(\theta + \gamma)}\right] \quad \text{………... (12)}$$

Let Inv_1, Inv_2, Inv_3 and Inv_4 be the total inventories in the intervals $0 \le t \le t_1, t_1 \le t \le t_2$ $t_2 \le t \le t_3$ and $t_3 \le t \le T$ respectively. Then

$$Inv_1 = \int_0^{t_1} I(t)dt = \left(\frac{a - D - \gamma S_0}{\theta^2}\right)\left(\theta.t_1 + e^{-\theta.t_1} - 1\right)$$

$$Inv_2 = \int_{t_1}^{t_2} I(t)dt$$

$$=\frac{1}{(\theta+\gamma)}\left(S_0-\frac{a-D}{(\theta+\gamma)}\right)\left\{1-e^{(\theta+\gamma)(t_2-t_1)}\right\}+\left(\frac{a-D}{\theta+\gamma}\right)(t_2-t_1)$$

$$Inv_3=\int_{t_2}^{t_3} I(t)dt$$

$$=\frac{1}{\theta+\gamma}\left[I(t_2)+\frac{D}{(\theta+\gamma)}\right]\left[1-e^{(\theta+\gamma)(t_2-t_3)}\right]-\left(\frac{D}{\theta+\gamma}\right)(t_3-t_2)$$

$$Inv_3=\int_{t_3}^{T} I(t)dt$$

$$=\left[\frac{S_0}{\theta}\right]+\left(\frac{D+\gamma S_0}{\theta^2}\right)\log\left(\frac{D+\gamma S_0}{D+(\gamma+\theta)S_0}\right)$$

By (12)

The values of θ, S_0 and $(D+\lambda S_0)$ must be such that $Inv_4>0$ is satisfied. Now the total deteriorated items I_d is

$$I_d=\theta\left\{\int_0^{t_1} I(t)dt+\int_{t_1}^{t_2} I(t)dt+\int_{t_2}^{t_3} I(t)dt+\int_{t_3}^{T} I(t)dt\right\}$$

$$=\theta(Inv_1+Inv_2+Inv_3+Inv_4)$$

Therefore the total demands in $(0,T)$ becomes $D_T=(a.t_2-I_d)$then the average profit during time $(0,T)$ takes the form

$$\pi(a,t_2)=\frac{1}{T}\left[(a.t_2-I_d)S_p-\left\{C_s+C_h(Inv_1+Inv_2+Inv_3+Inv_4)+\left(r+\frac{g}{a}+\alpha a\right)a.t_2\right\}\right]$$

Therefore we have to maximize $\pi(a,t_2)$ subject to the constraints

$$D+(\theta+\gamma)S_0-a<0$$
$$-Inv_1<0$$
$$-Inv_2<0$$
$$-Inv_3<0$$
$$-I(t_2)+S_0<0$$
$$-t_2+t_1<0$$

The condition

$$D+(\theta+\lambda)S_0-a<0 \Rightarrow 0<\frac{\theta.S_0}{a-D-\gamma S_0}<1$$

Which is the necessary for the value of t_1 in equation (6) to be real.The three conditions $-Inv_1<0, -Inv_2<0, -Inv_3<0$ ensure that Inv_1, Inv_2, Inv_3 must be positive. The condition $-I(t_2)+S_0<0$ implies that $I(t_2)$, the inventory level at time t_2 is higher than S_0. The condition $-t_2+t_1<0$ ensures that t_2 is greater than t_1.

VI. CONCLUSION

In this paper we developed the EPQ model with time dependent rate is decision variable on a volume flexible production policy. Shortages are allowed and backlogged. Our model is realistic due to present market situation.

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