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An Improved Design for the IIR Digital Fractional order Differential Filter using Indirect Discretization Approach

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ABSTRACT
In this paper, a novel design for improving the fractional order differential filter is put forward. And deals with continued fraction expansion (CFE) based indirect Discretization scheme for finding the rational approximation of fractional order differentiators By analyzing the frequency characteristic of typical fractional order differential filter, it can be seen that these kinds of differential filters have merits and demerits respectively and also could be complementary each other. So based on these features, three kinds of novel first order differential filters are constructed by the interpolated method. And then we choose a differential filter from these three kinds of filters which has much better frequency characteristic, also the improved IIR-type fractional order differential filter will be obtained by the method of continuous fraction expansion (CFE). The experiment result shows that the frequency response of the improved fractional order differential filter is more approximate to the ideal fractional order differential filter. And it also shows that the method put forward in the paper can improve the performance of the fractional order differential filter obviously under the premise of not increasing the structure complexity of the filter.

Keywords- *Rational order integrator, fractional calculus; digital differentiator; IIR filter; differential operator; continuous fractional expansion*

I. INTRODUCTION

Fractional calculus is known for describing a real object more accurately than the classical “integer order” methods asmost of these objects are fractional in nature The fractional order calculus is an old and modern topic. It had been proposed when the integral order calculus was produced, which had been involved and explored by many great mathematicians[1] such as Leibniz(1695), Euler(1738), Liouville(1850), Hardy and Littlewood(1925). But its physical meaning is undefined, which hinders the application of fractional order calculus. So the fractional order calculus is not applied in the engineering and technology widespread at present. Since Mandelbrot put forward the fractal dimension theory and analyzed the Brownian motion in the fractal dimension media by using the Riemann-Liouville fractional order calculus, the fractional order calculus had attracted the wide attention in many subjects especially in the Chemistry, Electromagnetic, Control Science, Material Science and Mechanics [2-6]. At present fractional order filter has been succeeded in applying in the fractional order controller, signal processing and image compression and processing and so on. The design and improving for the digital fractional order differential filter are becoming the hot issue in the field of fractional order calculus [7-11]. The design method of the digital differential filter is usually divided into two types, that is, the linear phase FIR filters and IIR filters. Considering the complexity factors when design the filter, the order of the FIR differential filters will be restrained and also the approximation effect of frequency response of the filter to the ideal frequency response is affected[12], so the IIR-type fractional order differential filter is considered to realize the fractional order operation in this paper. The common method used to expand the fractional-order includes the PSE (power series expansion) and CFE (continuous fraction expansion), and the method of CFE can make more effect on the functional approximation and has a faster convergence speed[13]. By analyzing the frequency response of typical operators, we can find that the fractional order differential filters based on the these kinds of typical differential filters have merits and demerits respectively and could be complementary each other, and we will get the frequency response operator more closed to the ideal fractional order differential operator by combining these kinds of typical operators.

II. THE TYPICAL IIR-TYPE FRACTIONAL ORDER DIFFERENTIAL FILTER

A. The IIR-type fractional order digital differential filter based on the Simpson operator

The Simpson differential operator is expressed as:

$$Hs(z) = \frac{3}{T}\frac{1-z-2}{1+4z} \tag{1}$$

So the transfer function of the Simpson fractional order differentiator can be expressed as

$$Gs(Z) = \left(\frac{3}{T}\frac{1-z^{-2}}{1+4z^{-1}}\right)v \tag{2}$$

In this paper we use the method of CFE to expand (2).We will briefly introduce the method of CFE which is used in the process of realizing the fractional order differential operator as follows: for the any function $D(z)$, we can use the continuous fraction form to express it, that is:

$$D(z) \cong a_0(z) + \cfrac{b1(z)}{a1(z)+\cfrac{b2(z)}{a2(z)+\cfrac{b3(z)}{a3(z)+\cdots\ldots\ldots}}} \tag{3}$$

Where the coefficients a_i ,b_i are rational functions or constants for the variable z . We can get the finite order approximating function just by the truncated operation. When T=0.001s, by using the method of CFE to expand (2) we can get the Simpson fractional order differential filter function $G^{v}_{Sn}(z)$ whose order is 0.5 easily, where v denotes differential order and n denotes the filter order. From the point of view of the error and its computational complexity, the order of the fractional order differential filter is relatively suitable to choose five. So the order of the filter mentioned in this paper is chosen as five

$$G_{s5}^{0.5} = \frac{54.77(z^5+5.86z^4+7.34z^3-6.31z^2-8.37z+2.63)}{z^5+7.87z^4+18.80z^3+6.505z^2-12.393z-1.4207} \tag{4}$$

B. The IIR-type fractional order digital differential filter based on the Rectangular operator

The Rectangular operator is expressed as:

$$Gs(Z) = \left(\frac{1}{T}\frac{1-z^{-1}}{1}\right) \tag{5}$$

So the transfer function of the fractional order differentiator based on the Rectangular operator can be expressed as:

$$G_R^v(Z) = \left(\frac{1}{T}\frac{1-z^{-1}}{1}\right)v \tag{6}$$

Here we also use the method of CFE to expand (6) and realize the finite order approximation for the function. When T=0.001s, the Rectangular fractional order differential filter be expressed as: function $G^{v}_{Sn}(z)$whose order is 0.5 is list out, where v denotes the differential order and n denotes the filter order.

$$G_{R5}^{0.5}(z) = \frac{31.62(z^5-2.75z^4+2.75z^3-1.202z^2+0.21z-0.010}{z^5-2.25z^4+1.75z^3-0.54z^2+0.056z} - 0.00097656 \tag{7}$$

C. The IIR-type fractional order digital differential filter based on the Tustin operator

The Tustin operator is expressed as:

$$G_T(z) = \frac{2}{T}\frac{1-z^{-1}}{1+z^{-1}} \tag{8}$$

So the transfer function of the fractional order differentiator based on the Tustin operator can be expressed as:

$$G_T^v = \left(\frac{2}{T}\right)^v\left(\frac{1-z^{-1}}{1+z^{-1}}\right) \tag{9}$$

Here we still use the method of CFE to expand (9) and realize the finite order approximation for the function. When T=0.001s, the Tustin fractional order differential filter function $G_{T5}^v(z)$whose order is 0.5 is list out, where v denotes the differential order and n denotes the filter order.

$$G_{T5}^{0.5} = \frac{44.72(z^5-0.5z^4-z^3+0.37z^2+0.187z-0.031}{z^5+0.5z^4-z^3-0.37z^2+0.18z+0.0313} \tag{10}$$

The figure 1(a) is the frequency characteristic curve of

0.5 order differential filter based on three typical operators which are Rectangular operator, Tustin operator and Simpson operator. According to the figure, we can find that the amplitude curves of three filters are basically consistent to the ideal amplitude in the low frequency region, but with the frequency increasing the errors will increase sharply especially in the high frequency region. According to the figure1(a) we can find the amplitude characteristic based on Rectangular operator is the best, but the phase characteristic is poorer than two other operators obviously. The advantage of the Tustin operator lies on its better phase characteristic and its phase characteristic is coincident to the phase curve of ideal frequency response in most regions. The Tustin operator and the Simpson operator have strong complementarily, because the two operators have better performance in the low frequency region. Although they have obvious errors in the high frequency region, the amplitude curves of both lie on the upper and lower bilateral is respectively. So the author believes that we can get a new operator whose frequency characteristic is much better by combining these three operators. We will introduce the deduction of new operator and its realization process in detail later.

III. THE NEW FRACTIONAL ORDER DIFFERENTIAL FILTER CONSTRUCTED BY COMBINING THREE TYPICAL OPERATORS

A. The fractional order differential filter based on the Rectangular operator and Tustin operator

To observe we find that the Rectangular operator and Tustin operator have the best amplitude-frequency characteristic and phase-frequency characteristic respectively, so we combine the two operators by the interpolated method and will get nearly ideal integrator. Because of the mutual inevitability between differentiation and integration, firstly, deducing the integral Operator $H_A(z)$, where A denotes integrator after combined and R denotes the Rectangular integrator and T denotes the Tustin integrator. The transfer function of integrator is given according to the rate that the Rectangular operator -Tustin operator ratio is 3:1. The transfer function of integrator is shown as follows:

$$H_A(z) = \frac{3}{4}\mathrm{H_R}(\mathrm{z}) + \frac{1}{4}\mathrm{H_T}(\mathrm{z}) \qquad (11)$$

By substitution of the corresponding transfer function, we can get

$$H_A(z) = \frac{3}{4}\frac{\mathrm{T}}{\mathrm{z}-1} + \frac{1}{4}\frac{\mathrm{T(z+1)}}{\mathrm{a(z-1)}} \qquad (12)$$

By simplifying the expression (12), we can get

$$H_A(z) = \frac{\mathrm{T(z+7)}}{8\mathrm{(z-1)}} \qquad (13)$$

Obviously, the zero of (13) is not included in the unit circle, so the zero z = -7 is mapped to z = -1/7 . Multiplied by seven we can make corresponding compensation for the amplitude and get the minimum phase integrator as follows:

$$H_A(z) = \frac{7T}{8}\frac{z+\frac{1}{7}}{(z-1)} \qquad (14)$$

By exchanging the numerator and denominator of (14) the differentiator is got, that is:

$$G_A(z) = \frac{8(z-1)}{7T(z+\frac{1}{7})} \qquad (15)$$

whose corresponding fractional order differential Operator $G_A{}^{v}(z)$ is:

$$G_A^{v} = \left(\frac{8}{7T}\frac{z-1}{z+\frac{1}{7}}\right) \qquad (16)$$

When T=0.001s, the IIR-type fractional order differential filter function whose order is 0.5 is implemented by the new operator A.

$$G_{A5}^{0.5} = \frac{33.8(z^5-2.4z^4+2z^3-0.6122z^2+0.03z+.004}{z^5-1.857z^4+1.0204z^3\pm0.122z^2-0.021z+0.0014} \qquad (17)$$

B. The fractional order differential filter based on the Tustin operator and Simpson operator

Likewise, to observe we find that both Tustin operator and Simpson operator exist error in the high frequency region, but their amplitude curves lie on the upper and lower bilateralis respectively, so we intend to combine the two operators by the interpolated method and then get the filter being much better performance. The transfer function of integrator $H_B(z)$ improved is given according to the rate that the Tustin operator - Simpson operator ratio is 2:3.

$$H_B(z) = \frac{2}{5}H_T(z) + \frac{3}{5}Hs(z) \quad (18)$$

By substitution of the corresponding transfer function and simplifying it, we can get

$$H_B = \frac{2T}{5}\frac{z^2+3z+1}{z^2-1} \quad (19)$$

By exchanging the numerator and denominator of the expression (19) to get corresponding differentiator, that is

$$G_B = \frac{5}{2T}\frac{z^2-1}{z^2+3z+1} \quad (20)$$

The zeros of (19) are $\boldsymbol{r1 = (-3+\sqrt{5})/2}$ and $r2 = (-3-5\sqrt{5})/2$. Comparing $\mathbf{r}_1$ with r_2, it is easy to knowT_1 is the reciprocal of r_2 andr_2 is not included in the unit circle. To construct the minimum phase system, r_2 is mapped to r_1. At the same time, to keep the amplitude invariant we introduce the compensation factor - r_2 and get the integral operator as follows:

$$H_B(z) = \frac{-2Tr_2}{5}\frac{(z-r_1)^2}{z^2-1} \quad (21)$$

In addition, the differential operator improved is

$$G_B(z) = \frac{-5r_1}{2T}\frac{z^2-1}{(z-r_1)^2} \quad (22)$$

whose corresponding fractional order differential operator $G_B^{\,v}(z)$ is:

$$G_B^v(z) = \left(\frac{-5r_1}{2T}\frac{z^2-1}{(z-r_1)^2}\right)^v \quad (23)$$

The poles of integral operator (21) are 1 and -1 which are all on the circle, so they do not satisfy the system stability. But by the method of CFE to expand the expression (23) and then truncating the expression expanded, the two poles can be included in the unit circle.

When T=0.001s, the IIR-type fractional order differential filter function $G_B{}^v{}_n(z)$ whose order is 0.5 is implemented by the new operator B

$$G_{B5}^{0.5}(z) = \frac{30.9(z^5-0.0016z^4-1.4994z^3+0.0973z^2+0.5254z-0.0818)}{z^5+0.3804z^4-z^3-0.2853z^2+0.1875z+0.0238} \quad (24)$$

C. The fractional order differential filter based on the Rectangular operator and Simpson operator

The new operator can be formed by combining the Rectangular operator and Simpson operator similarly. The transfer function of integral operator $H_C(z)$ improved is given according to the rate that the Rectangular operator -Simpson operator ratio is 5:3.

$$H_C(z) = \frac{5}{8}H_R(z) + \frac{3}{8}H_S(z) \quad (25)$$

By substitution of the corresponding transfer function and simplifying it, we can get

$$H_C(z) = \frac{6T}{8}\frac{z^2+3/2z+1/6}{z^2-1} \quad (26)$$

The zeros of expression (26) are $r1 = (-9-\sqrt{57}/12 \; and \; r2 = (-9\sqrt{57})/12$, and the zero r_2 is not included in the unit circle. To construct the minimum phase system, the zeror_2 is mapped to$1/r_2$. At the same time, to keep the amplitude invariant we introduce the compensation factor r_2 and get the integral operator as follows

$$H_C(z) = \frac{-3Tr_2}{4}\frac{(z-r_1)(z-1/r_2)}{z^2-1} \quad (27)$$

In addition, the differential operator improved is

$$G_C(z) = \frac{-4}{3Tr_2}\frac{z^2-1}{(z-r_1)(z-1/r_2)} \quad (28)$$

The poles of expression (27) are 1 and -1 which are all on the circle, so they do not satisfy the system stability. But by the method of CFE to expand the expression (23) and then truncating the expression expanded, the two poles can be included in the unit circle.

When T=0.001s, the IIR-type fractional order differential filter function $G_{cn}^{v}(z)$ whose order is 0.5 is implemented by the new operator C

$$G_{C5}^{0.5}=\frac{31.09(z^5+0.1486z^4-1.6961z^3+0.0138z^2+0.7061z-0.1329)}{z^5+0.5716z^4-1.1788z^3-0.4052z^2+0.3186z+0.0047} \quad (29)$$

Figure 1(b) is the frequency response curve of fractional order differential filter based on three new operators. And according to the curve the new operator A is superior to the operator B and C in the frequency characteristic and its amplitude characteristic is basically close to the ideal frequency response curve from the low frequency region to high frequency region. The curve show that the phase characteristic of new operator A is approximate linearly increasing with the increasing of frequency, so we can introduce the fractional order delay filter to improve the phase characteristic furthermore.

IV. CONCLUSIONS

This paper makes deep analysis on the design and implement for the IIR-type digital fractional order differential filter from the point of view of frequency region. The implement for IIR-type digital fractional order differential filter has two important procedures: firstly, we should find the suitable differential operator, because the similarity between the frequency response of selected operator and ideal fractional order differential filter has the direct effect on the implement of chosen filter; secondly, we should use suitable expansion method to transform the transfer function from the integral order form to the fractional order form, and in this paper we choose the method of CFE, which is widely used and has well performance. In the paper we get an operator which is more close to the ideal frequency response of operator by combining some kinds of typical operators using interpolated method, and realize the IIR-type digital fractional order differential filter by the method of CFE. By the contrastive analysis of frequency response of new operator, it is easy to know that the performance of fractional order differential filter can be improved obviously.

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