Design and Performance Analysis of IIR Filters for Reducing Signal Noise

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ABSTRACT

During the transmission of signal, some noises are introduced due to channel non linearties and various impairments. This deteriorates the original signal. Therefore it becomes essential to separate these noises from the original signal. This is the simplest aim of signal processing to reduce these noises. In this paper Butterworth low pass filter which is the first classical analog filter and Elliptic low pass filter which is the last classical analog filter are used to reduce the noise and then a comparative study is made between the performances of the two filters using MATLAB simulation. The MATLAB results show that Elliptic filter removes the noise more effectively than the Butterworth filter.

***Key** Words: Butterworth, DSP, Elliptic, IIR.*

1. INTRODUCTION

Digital signal processing is the most power technology and fastest growing fields in modern electronic world. They are broadly used in areas where information are floated in digital form or controlled by using digital processor. DSP has a core impact on the areas like telecommunications, medical imaging, radar & sonar, high fidelity music system, digital television, digital audio and instrumentation, compression of data for storage and transmission, and oil prospecting. Each of these areas has developed a *deep* DSP technology, with its own algorithms, mathematics, and specialized techniques. Digital signal processing means the signals are represented in digital form and we use digital processor to analyze, modify, or extract information from signals. By nature most of the signals are analog in form and vary continuously with time. The signals which are to be used for signal processing are taken from analog signals and are then converted into digital form by sampling and quantization.

The primary use of DSP is to reduce interference, noise, and other undesirable components in acquired data, to obtain the frequency spectrum, or to transform the signal into a form that is more suitable and convince to process. DSP has captured most of the areas where analog methods were previously used and in entirely new applications, like digital multimedia, where implementation of analog methods are difficult and sometimes impossible. Although the DSP approach requires more steps than analog signal processing, there are important benefits to working with signal in digital form. The primary advantages of analog signal processing are speed and cost. The most important benefits of DSP is the inherent flexibility associated with a software implementation.DSP gives higher performance, less drifting with age and with change in environment conditions and linear phase responses and therefore manual calibration is not required. The fundamental DSP operations require convolution, correlation, filtering, transformation, and modulation. Due to the availability of more sophisticated and specialized commercial signal processing software, there is now wide flexibility and controllability to analyze and develop more complicated signal processing application in a rapid and reliable fashion [1].

Filters are considered the basic element of all signal processing and telecommunication system. Filters are commonly used in signal processing and communication systems in applications such as channel equalization, noise reduction, radar system, audio and video processing, and biomedical signal, processing and analysis of economic and financial data. . A filter is basically a frequency- selective device that permits us to shape the magnitude or phase response in a systematic manner. Filters are classified on the basis of their operating signals as analog or digital. Digital filters are very important part of DSP. In fact its extraordinary performance in processing makes DSP so popular. Filters have basically two uses: signal separation and signal restoration. Signal separation is required when the signal has been contaminated with interference, noise or other signals. In filters the approximation step is utilized for generating a transfer function that satisfies the desired specification either in frequency or in time-domain. In approximation step process, the coefficient of the transfer function is carried out to a high degree of precision [2].

2. BUTTERWORTH FILTER DESIGN

The Butterworth filter provides a maximally flat response. The advantage of the Butterworth filter is that the calculations are somewhat simpler than those for other forms of filter. This simplicity combined with a level of performance make it an adequate technique for many areas of electronics from RF to audio active filters. Using the equations for the Butterworth filter, it is relatively easy to calculate and plot the frequency response as well as working out the values needed. Butterworth contains a number of useful qualitative properties. The magnitude response decreases monotonically that starts from $A_0 = 1$. For higher frequencies the asymptotic attenuation of an nth order filters is 20ndB per decade [3]. The most important property of Butterworth filters is that the first 2n-1derivative of the squared magnitude are zero at f=0. That means pass band is maximally flat. The response is monotonically decreasing from the specified cut off frequencies. The maximum gain occurs at Ω= 0 and it is |H (0)|= 1. Half power frequency, or 3db down frequency, that corresponds to the specified cut off frequencies. .Butterworth filter achieve its flatness at the expense of a relatively wide transition region from pass band to stop band with average transient characteristics [3].

The ideal frequency response, referred to as a “brick wall” filter, and the standard Butterworth approximations, for different filter orders are given below [4]

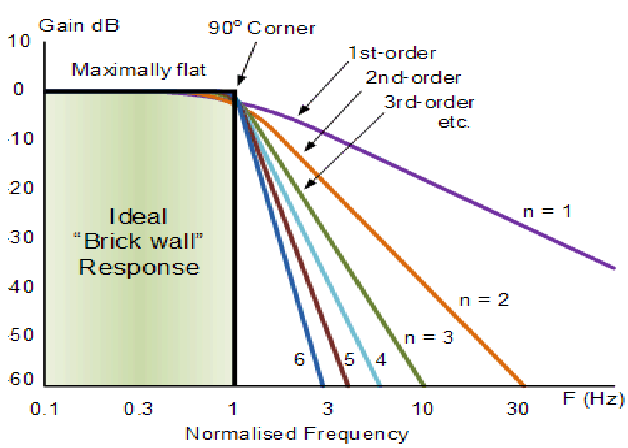

Fig.1 Frequency Response for a Butterworth Filter

From the figure it is observed that higher the Butterworth filter order, the higher the number of cascaded stages there are within the filter design, and the closer the filter becomes to the ideal “brick wall” response. This filter is completely defined mathematically by two parameters i.e. cut off frequency and number of poles. The frequency response of the “nth”order Butterworth filter can be represented as

$$|H(f)| = \frac{1}{\sqrt{1+\left(\frac{f}{f_c}\right)^{2n}}} \qquad (1)$$

Where n represents the order of the filter (number of poles), $|H(f)|$ is the magnitude in stopband frequency, fc is the cutoff frequency [3, 5, 7].

3. ELLIPTIC FILTER DESIGN

Elliptic filters contain equiripple characteristics both in the pass-band and the stop-band. The elliptic filters are optimal in terms of a minimum width of transition band; they provide the fastest transition from the band-pass to the bandstop. Elliptic filters are also well known as Cauer filters or Zolotarev filters. Elliptic filters achieve the smallest filter order for the same specifications, or, the narrowest transition width for the same filter order, as compared to other filter types. On the negative side, they have the most nonlinear phase response over their pass band. Elliptical function filters contain zero as well as poles in the transfer function, this result in infinite rejection at stopband frequency near cut-off.

The magnitude-square response of Elliptic filter is given by

$$|H_a(J\omega)|^2 = \frac{1}{1+\epsilon^2 U_N^2\left[\frac{\Omega}{\Omega_c}\right]} \qquad (2)$$

Where N is the order,∈ is the Pass band ripple and $U_N(\cdot)$ is the N^{th} order Jacobian Elliptic function. even though the analysis of equation (1) is difficult, the order calculation formula is very compact [4,5].It is given by

$$N=\left[\frac{K(k)K\left(\sqrt{1-k_1^2}\right)}{K(k_1)K\left(\sqrt{1-k^2}\right)}\right] \qquad (3)$$

Where $k=\frac{\Omega_p}{\Omega_s}$

$$k_1 = \frac{\epsilon}{\sqrt{A^2-1}} \qquad (4)$$

$$K(x)=\int_0^{\pi} \frac{d\theta}{\sqrt{1-x^2\sin\theta}} \qquad (5)$$

Equation (5) is the complete integral of first kind. MATLAB provides the function ellipke to numerically compute the above integral. MATLAB provides a function called $[z, p, k] = ellipap(N, R_p, A_s)$ to design a normalised elliptic analog prototype filter of the order N, Passband ripple RP , Stopband attenuation AS , it gives pole and zeros and constant K after designing analog prototype filter it is required to convert it into digital filter using any method.

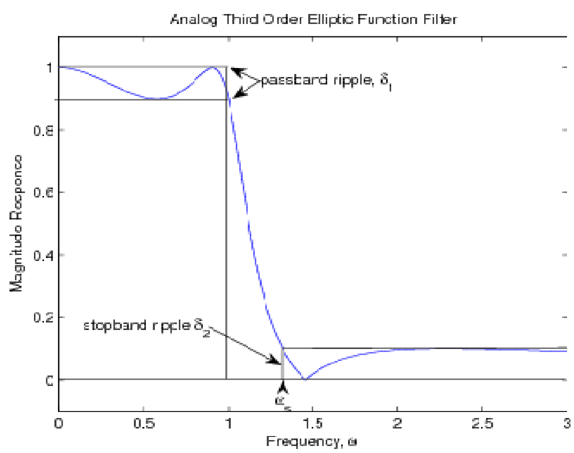

Fig.2 Analog third order Elliptic filter

Elliptic filters are more complex and critical to analyze and design than other type of analog filters. Calculation of zeros and poles of an Elliptical filter involves the iterative solution of nonlinear equations. Elliptic filters always meet the passband specification exactly as long as the ripple factor ∈ is chosen as in equation [3].

$$\in = \left[\left(1-\delta_p\right)^{-2}-1\right]^{\frac{1}{2}} \quad (5)$$

Where δ_p is the desired passband ripple.

4. MATLAB Based Design Simulation

In the MATLAB, we generate a cosine signal and add the noise in the original signal. This results a noisy signal and then pass it, first through the Butterworth filter of different order (N = 2, 10, 20), cut-off frequency $\Omega_c = 0.3$ and analyze the responses. The noisy signal is then passed through the Elliptic filter with same orders and having Passband ripple $R_p = 0.5$ .and stopband attenuation $A_S = 40$ and $then$ analyze the response. Fig.3 shows the noisy signal, which is to be filtered, in time domain. Fig.4 shows the response of the Butterworth filter at N=2, 10, 20 in time domain with same value of $\Omega_c = 0.3$.Here it is found that with increase in order, the smoothness of the signal increases and it is better in case of N=20. Fig.5 shows the magnitude spectrum response at N=2, 10, 20.Here, it is found that the flatness of the signal increases with increase in number of order and is maximum at N=20. In fig. 6 a magnitude response comparison is made between the two filters and found that the Elliplic filter shows more sharpness than Butterworh filter with the increase in number of order ,that is at N=20

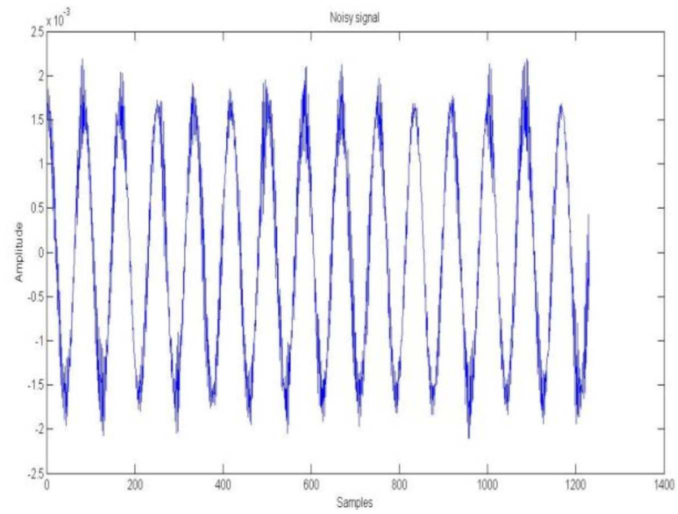

FIG.3 NOISY SIGNAL

Fig.4 Butterworth Filtered signal at N=2, 10, 20 in time domain.

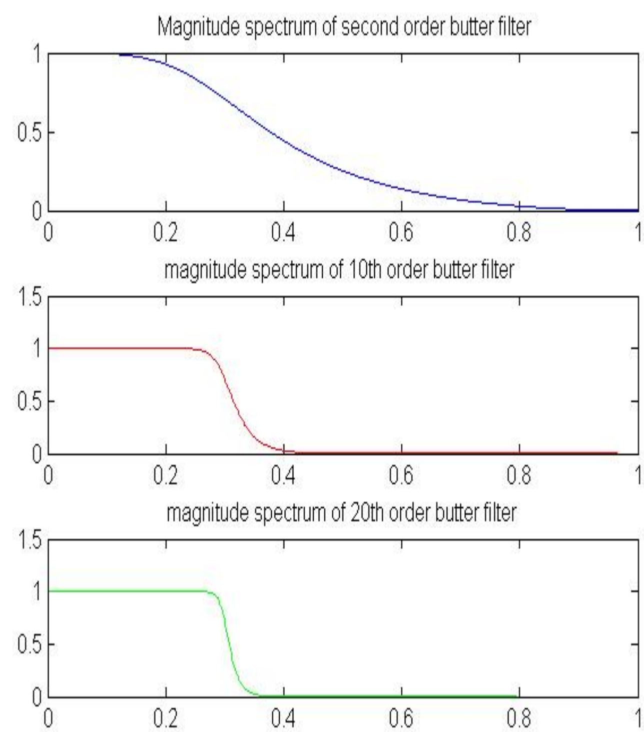

Fig.5 Frequency response of Butterworth, N=2, 10, 20

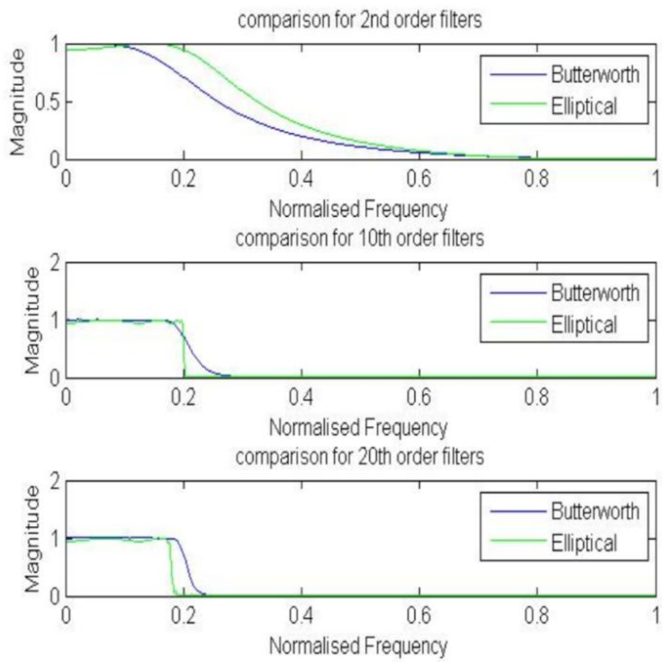

Fig.6 Comparison of Butterworth, Elliptic filter at N= 2, 10, 20.

5. CONCLUSION

In this paper noisy signal have been analyzed and passed though Butterworth filter with different values of order N and concluded that higher values of N results more smoothness in the signal. We also compared the amplitude response of the Butterworth filter with the Elliptic filter and concluded that Elliptic filter response results more sharpness but has a ripple in the magnitude and in Passband frequency compared with Butterworth filter.

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