# Systems: The Foundation

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## ABSTRACT
Every minute particle is performing some process which signifies a system, therefore it is very necessary and required to understand input to system, process of the system and output of the system, so that a uniform analysis can be made to develop and understand various process occurring in surrounding(internal and external). The basic knowledge about the systems is very important for understanding the behaviour of output obtained for a given input. The input output relationship can be totally described by the proper knowledge of the system and its behaviour. From the prior information about the system it becomes easy to design the desired system for a particular function. This article describes with proper examples the basic types and functions of systems used for input output mapping of signal in various fields.

***Index Terms***— *classification, time invariant, causal, stable, linear, static, dynamic, invertible, distributed parameters systems.*

## I. INTRODUCTION TO SYSTEMS

Attempting a formal definition of a system is a tedious exercise in avoiding circularity, so we will abandon precision and rely on the intuition that develops from examples. Thus the definition follows given conditions:

- Physical systems in the broadest sense are an interconnection of components, devices, or subsystems.
- A system can be viewed as a process in which input signals are transformed by system and resulting in other signals as output.
- A system can be viewed as a function that maps signals into signals.
- We represent a system "S" with input signal x(t)/ x[n] and output signal y(t)/ y[n] by a box labelled as shown

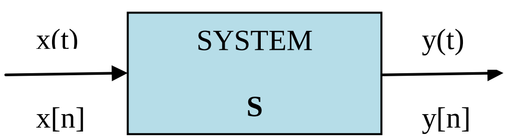


Fig. 1: System Representation

The output signal at any time t can depend on the input signal values at all times. We use the mathematical notation

**y(t)=S(x)(t)** for continuous time systems
**y[n]=S(x)[n]** for discrete time systems

to emphasize this fact.

## II. EXAMPLES OF SYSTEMS

- The running integral is an example of a system. A physical interpretation is a capacitor with input signal the current *i(t)* through the capacitor, and output signal the voltage *v(t)* across the capacitor. Then we have, assuming unit capacitance,

$$v(t) = \int_{-\infty}^{t} i(\tau)\, d\tau$$

In this case, the output at any time $t_1$ depends, on input values for all t. Specifically, at any time $t_1$, is the accumulated net area under *i(t)* for $-\infty < t \leq t_1$.

- The electrical circuit is a example of system which comprises various system components such as resistors, capacitors, inductors, transistors, and so on. Voltages and currents in the circuit are signals.

- The wheel suspension is a system. It comprises various system components such as wheel, tyre, spring, shock absorber, and so forth. The position and velocity of various system components are signals.

## III. CLASSIFICATION OF SYSTEMS

In broad sense systems are classified as

- Continuous time and discrete time systems,
- Distributed parameters and lumped parameters systems,
- Static and dynamic systems,
- Invertible and inverse systems,
- Causal and non causal systems,
- Linear and non linear systems,
- Time invariant and time varying systems,
- Stable and unstable systems.

## *A.* CONTINUOUS TIME AND DISCRETE TIME SYSTEMS

A continuous time system is a system in which continuous time signals are applied and results in continuous time output signals. Such a system is represented as

$$x(t) \rightarrow y(t)$$

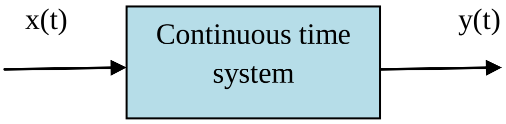


Fig. 2: Continuous Time System Representation

where x(t) is input signal and y(t) is output signal.

A discrete time system is a system in which discrete time signals are applied and results in discrete time output signals. Such a system is represented as

$$x[n] \rightarrow y[n]$$

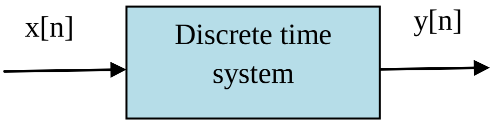


Fig. 3: Discrete Time System Representation

where x[n] is input signal and y[n] is output signal.

*Examples Of Continuous Time And Discrete Time Signals*

- Consider the waveforms shown for sine function for a continuous time signal and a discrete time signal

```
t=0:.001:2*pi;
x=sin(t);
plot(t,x);
xlabel('time');
ylabel('amplitude');
title('continous time signal');
```

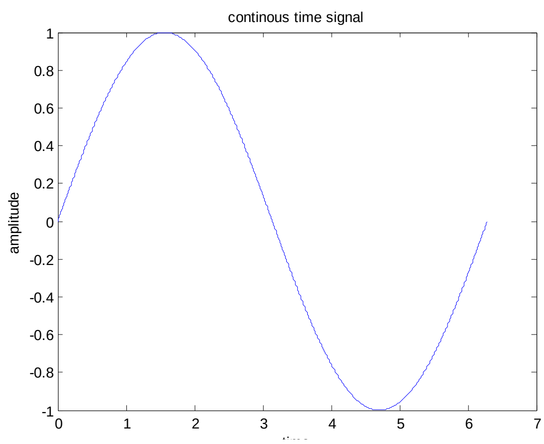


Fig. 4: Example Continuous Time System

```
n=0:.5:2*pi;
x=sin(n);
stem(n,x);
xlabel('time instants');
ylabel('amplitude');
title('discrete time signal');
```

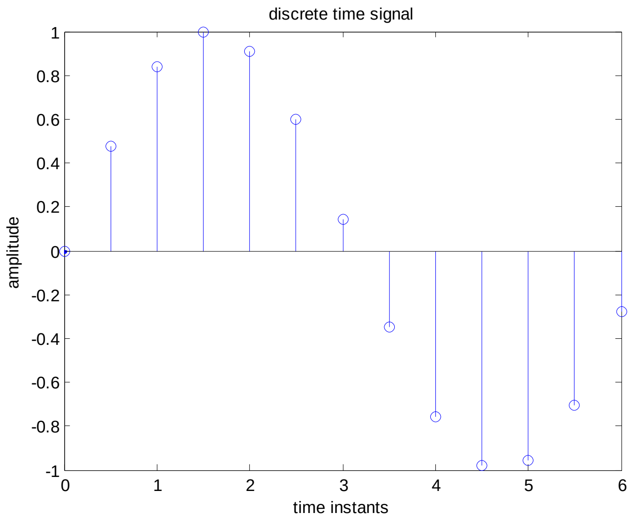


Fig. 5: Example Discrete Time Sequence

## *B.* DISTRIBUTED PARAMETERS AND LUMPED PARAMETERS SYSTEMS

In the most general sense, all physical systems contain distributed parameters because of the physical size of the components. For example, resister is a distributed parameter system because its resistance is distributed throughout its volume. The distributed parameter systems are modelled with partial differential equations for continuous time systems or with partial difference equations for discrete time systems.
If the size of the component is large with respect to the wavelength of the highest frequency component present in the signals associated with it is called a ***distributed parameter system***.

In the action of system occurring at a point and the size of the component is small with respect to the wavelength of the highest frequency component present in the signals associated with it and it is true for all components in the system, then the system is called ***lumped parameter systems***.
The lumped parameter systems are modelled with ordinary differential equations for continuous time systems or with ordinary difference equations for discrete time systems.

*Examples of Distributed Parameters and Lumped Parameters Systems*

50 Hz electric power system is an example of a distributed parameters systems as well as lumped parameters systems depends upon the wavelength of the signal and size of the component. If wavelength of the signal is 6000 Kms, then a electrical system inside a building can be treated as lumped parameters system and same system is treated as distributed parameter system for long distance transmission line.

## *C.* STATIC AND DYNAMIC SYSTEMS

On the basis of memory requirement, systems are categorised as static (memory less) or dynamic (system with memory) systems.
Static systems are also called *memory less systems.* Physically these systems contain no energy storage elements. The equation relating its output signal to its input signal does not contain any derivative, integrals, or signal delays.
A system is memory less if the output value at any time t depends only on the input signal value at that same time, t. *A memory less system is causal, though the reverse is untrue.*
Let us take the ***example***

$$y[n] = (2x[n] - x^2[n])^2$$

is memory less, as the value of *y[n]* at any particular time $n_0$ depends only on the value of x[n] at that time.

*Other examples of memory less systems are*

$$y(t) = 2x(t)\ ,\ y(t) = x^2(t)\ ,\ y(t) = te^{x(t)}$$

A dynamic system or *system with memory* is a system with an output signal that at every specified time depends on the value of the input signal at both the specified time and at other times. These systems have one or more energy storage elements. Input output relationship of a dynamic continuous time system is described by its differential equation and by its difference equation for discrete time system.

*Examples of systems with memory are*

- A capacitor is an example of continuous time system with memory and its input output relation is given by

$$v(t) = \frac{1}{C}\int_{-\infty}^{t} i(\tau)d\tau$$

Where *v(t)* is the output voltage
*i(t)* is the input current
and *C* is the capacitance of the capacitor.

- An accumulator is an example of a discrete time system with memory and its input output relationship given by

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]$$

- A delay element is also an example of dynamic system and its input output relationship is given by

$$y[n] = x[n-1]$$

## *D.* INVERTIBLE SYSTEMS AND INVERSE SYSTEMS

A system is invertible if the input signal can be uniquely determined from knowledge of the output signal.

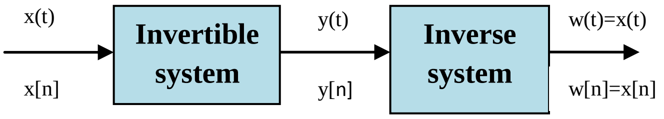


Fig. 6: Invertible System Representation

*Examples of invertible systems are*

$$y(t) = x^3(t)\,,\ y(t) = 3x(t-2) + 4t$$

The thoughtful reader will be justifiably nervous about this definition. Invertibility of a mathematical operation requires two features: the operation must be ***one-to-one*** and also ***onto***. Since we have not established a class of input signals that we consider for systems, or a corresponding class of output signals, the issue of "onto" is left vague. And since we have decided to ignore or reassign values of a signal at isolated points in time for reasons of simplicity or convenience, even the issue of "one-to-one" is unsettled.
Determining invertibility of a given system can be quite difficult. Perhaps the easiest situation is showing that a system is not invertible by exhibiting two legitimately different input signals that yield the same output signal.
If a system is invertible, then an inverse system exists. The cascading of an invertible system and its inverse system is equivalent to the identity system.

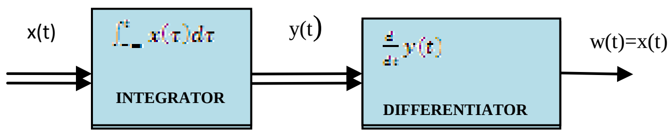


Fig. 7: example of invertible system

*Examples of invertible systems and inverse systems*

- For example, $y(t) = x^2(t)$ is not invertible because constant input signal of *x(t)=1* and *x(t)=-1, for all t,* yield identical output signals.
- As another example, the system

$$y(t) = \frac{d}{dt}x(t)$$

is not invertible since $\tilde{x}(t) = x(t) + 1$ yields the same output as the signal *x(t)* yields.

- As a final example,

$$y(t) = \int_{-\infty}^{t} x(\tau)d\tau$$

is invertible by the fundamental theorem of calculus:

$$\frac{d}{dt}\int_{-\infty}^{t} x(\tau)d\tau = x(t)$$

Inverse system for this system is

$$w(t) = \frac{d}{dt}y(t)$$

But the fact remains that technicalities are required for this conclusion. If two input signals differ only at isolated points in time, the output signals will be identical, and thus the system is not invertible if we consider such input signals to be legitimately different.

## *E.* CAUSAL AND NON-CAUSAL SYSTEMS

A system is *causal* if the output signal value at any time t depends only on input signal values for times no larger than t. Or in other words, a system is causal if the output signal value at any time t depends only on input signal at the present time and in the past. In such systems the system output does not anticipate future values of the input. Consequently, if two inputs to a causal system are identical up to some point $t_0$ or $n_0$, the corresponding outputs must also be equal up to this same time. *All memory less systems are causal.*

A system is called non-causal if its output at any given time depends on the input at future time. Some of the non-real systems are non-causal. Image processing systems are non-causal.

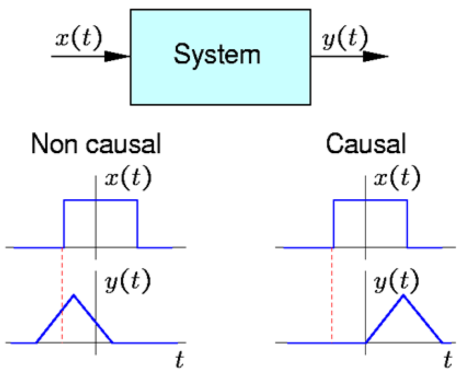


Fig. 8: Causal System Representation

*Observations on causality:*

- A system is causal if the output does not anticipate future values of the input, i.e., if the output at any time depends only on values of the input up to that time.
- All real-time physical systems are causal, because time only moves forward. Effect occurs after cause. (Imagine if you own a non-causal system whose output depends on tomorrow's stock price.)
- Causality does not apply to spatially varying signals. (We can move both left and right, up and down.)
- Causality does not apply to systems processing recorded signals, e.g. taped sports games vs. live broadcast.

*Examples of causal and noncausal systems:*

- A resister described by $v(t) = Ri(t)$ is a causal continuous time system as the output $v(t)$ , *i.e.*, the voltage depends only on the input current $i(t)$ at present time.
- Delay element described by $y[n] = x[n-1]$ is a causal discrete time system because output $y[n]$ depends only on past input $x[n-1]$ , not on future input.
- Systems defined by $y[n] = x[n] - x[n+1]$ and $y(t) = x(t+1)$ are non-causal systems because output at time *t* or *n* depends on the input at future.

## *F.* LINEAR AND NON-LINEAR SYSTEMS

A system is linear if for every pair of input signals $x_1(t), x_2(t)$ with corresponding output signals $y_1(t), y_2(t)$ the following holds.

*"For every constant b, the response to the input signal* $x(t) = bx_1(t) + x_2(t)$ *is* $y(t) = by_1(t) + y_2(t)$*."*

(This is more concise than popular two-part definitions of linearity in the literature. Taking b = 1 yields the *additivity* requirement that the response to $x(t) = x_1(t) + x_2(t)$ be $y(t) = y_1(t) + y_2(t)$. And taking $x_1(t) = x_2(t)$ gives the *homogeniety* requirement that the response to $x(t) = (b+1)x_1(t)$ should be $y(t) = by_1(t) + y_2(t)$ for any constant b.)

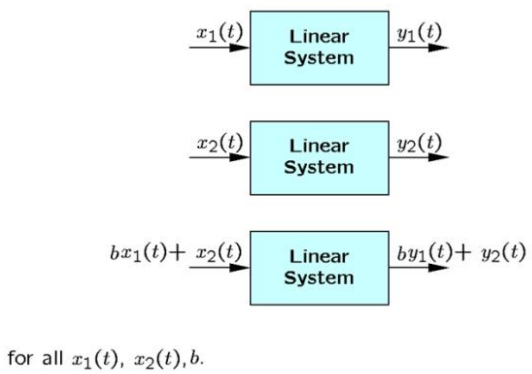


Fig. 9: Linear System Representation

A non-linear system (continuous time/ discrete time) is a system which does not satisfy the above two properties of *additivity* and *homogeneity*.

*Examples of linear and non-linear systems*

- The example of linear system is ***y(t)=tx(t).*** let us take the output for $x_1(t), x_2(t)$ as $y_1(t), y_2(t)$ respectively, then we have

$$y_1(t) = tx_1(t)$$
$$y_2(t) = tx_2(t)$$

Let's check for *additivity*,

If we apply x(t)= $x_1(t) + x_2(t)$ then from the given equation we get

$$y(t) = t(x_1(t) + x_2(t)).$$

And from $y(t) = y_1(t) + y_2(t)$ we get

$$y(t) = y_1(t) + y_2(t)$$
$$= tx_1(t) + tx_2(t)$$
$$= t(x_1(t) + x_2(t))$$

Hence additive property stands.

Lets check for the *homogeneity.*

If we apply input x(t)= $(b+1)x_1(t) = (b+1)x_2(t)$ then from the equation we get

$$y(t) = (b+1)tx_1(t) = (b+1)tx_2(t)$$

and from definition $y(t) = y_1(t) = y_2(t)$

$$y_1(t) = (b+1)tx_1(t)$$
$$y_2(t) = (b+1)tx_2(t)$$

Hence *homogeneity* also stands.

- Other examples of linear systems are:

$$\boldsymbol{y(t) = e^t x(t),}$$
$$\boldsymbol{y(t) = 3x(t-2),}$$
$$\boldsymbol{y(t) = \int_{-\infty}^{t} x(\tau)d\tau}$$

- The example of linear system is ***y(t)=x²(t).*** let us take the output for $x_1(t), x_2(t)$ as $y_1(t)$, $y_2(t)$ respectively, then we have

$$y_1(t) = x_1^2(t)$$
$$y_2(t) = x_2^2(t)$$

Let's check for *additivity*,

If we apply x(t)= $x_1(t) + x_2(t)$ then from the given equation we get

$$y(t) = (x_1(t) + x_2(t))^2.$$
$$= x_1^2(t) + x_2^2(t) + 2x_1(t)x_2(t)$$

From definition of additivity we get

$$y(t) = y_1(t) + y_2(t)$$
$$y(t) = x_1^2(t) + x_2^2(t)$$

Since the equation from definition differs from that of from equation given so the system is not additive.

Let's check for homogeneity.

If we apply input x(t)= $(b+1)x_1(t) = (b+1)x_2(t)$ then from the equation we get

$$y(t) = ((b+1)x_1(t))^2 = ((b+1)x_2(t))^2$$

and from definition $y(t) = y_1(t) = y_2(t)$

$$y_1(t) = ((b+1)x_1(t))^2$$
$$y_2(t) = ((b+1)x_2(t))^2$$

Hence *homogeneity* stands.

The system is not linear as it does not hold additive property.

- Other examples of non-linear systems are:

$$\boldsymbol{y(t) = \cos x(t), y(t) = 1 + x(t),}$$
$$\boldsymbol{y(t) = \int_{-\infty}^{t} x^2(\sigma)d\sigma}$$

## G. TIME INVARIANT AND TIME VARYING SYSTEMS

A system is called time-invariant if a time shift (delay or advance) in the input signal causes the same time shift in the output signal. Thus, the system is time-invariant if any shift in input causes same shift in output. A system is said to be time invariant if for a system

***If y(t)=F(x(t))***

then ***y(t-t₀)=F(x(t-t₀))***

The system which varies with time is said to be time varying systems.

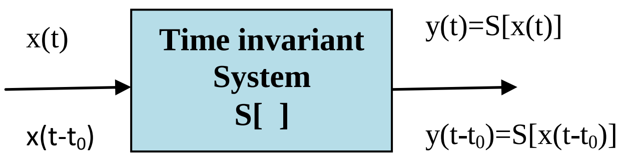


Fig. 10: Representation of Time Invariant System

*Steps to find out step invariant systems*

- For a given system find output response for shifted input, i.e. $\boldsymbol{y(t,t')=S[x(t-t_0)]}$, by replacing all $\boldsymbol{x(t)}$ by its time shifted version $\boldsymbol{x(t-t_0)}$
- For the same system find shifted output response, i.e. $\boldsymbol{y(t-t_0)}$ by replacing all $\boldsymbol{t}$ by $\boldsymbol{(t-t_0)}$
- If $\boldsymbol{y(t,t')= y(t-t_0)}$ then system is time invariant else time varying system.

*Examples of time invariant and time varying systems*

- Consider a system with input *x[n]=sin[n]* and output *y[n]=nx[n]*.

  Shifted input→*x[n-1] = sin[n-1]*……shifted by 1

  Shifted output→*y[n-1] = (n-1)x[n-1]*…….shifted by one

  Output as function of shifted input→*y[n,1] = nx[n-1]* ………just by shifting input only

  For n = π/2 the program computes the value of *y1[n ]= y[n,1] = nx[n-1] = nsin[n-1] = π/2sin[π/2-1] = 0.3084* by replacing *x[n]* by *x[n-1]*.
  *y2[n] = y[n-1] = (n-1)x[n-1 ]= (n-1)sin[n-1] = (π/2-1)sin[π/2-1] = 0.8487*

  since *y[n,1] ≠ y[n-1]* therefore system is *Time Variant*.

  *Program*

```
    n=(pi/2);
    y=n*sin(n);
    subplot(3,1,1);
    stem(n,y);
    xlabel('time instants[n]');
    ylabel('amplitude');
    title('y=nsin[n]');
  y1=(n-1)*sin(n-1);
  subplot(3,1,2);
  stem(n,y1);
  xlabel('time instants[n]');
  ylabel('amplitude');
  title('y1[n]=y[n-1]');
  y2=n*sin(n-1);
  subplot(3,1,3);
  stem(n,y2);
  xlabel('time instants[n]');
  ylabel('amplitude');
  title('y2[n]=f(x[n-1])');
```

- Consider the system with output *y[n]=cos(x[n])* with *x[n]* as input.
  *y1[n] = y[n,1] = cos(x[n-1])* by replacing *x[n]* by
  *x[n-1]*.
  *y2[n] = y[n-1] = cos(x[n-1])*
  since *y[n,1] = y[n-1]* therefore system is *Time Invariant*.
- Other examples of time variant system
  *y[n] = x[-n]*,
  $y(t) = x(t)\cos\omega_0 t$,
  *y(t) = x(t) + tx(t+1)*

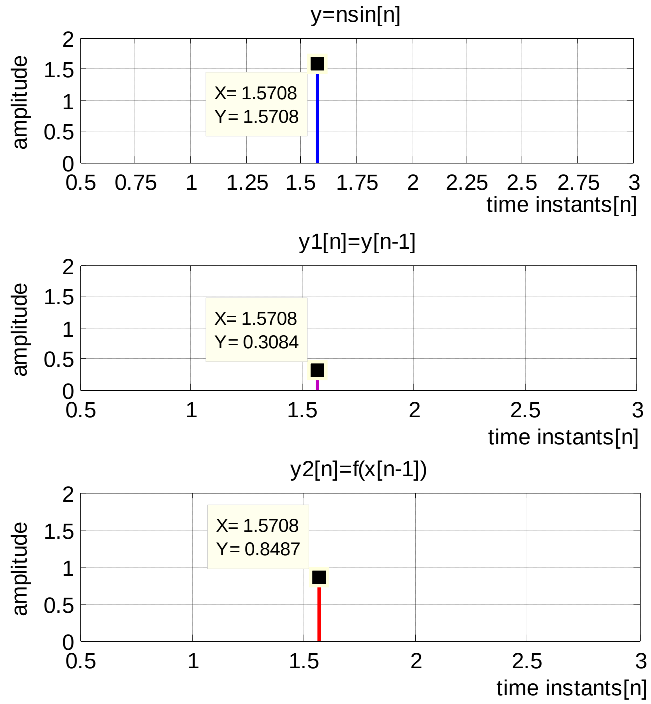


Fig. 11: example of time variant system (a) shows output at n=pi/2, (b) shows shifted output, (c) shows output when input is shifted

### *H.* STABLE AND UNSTABLE SYSTEMS

A **system** is said to be BIBO (bounded input bounded output) **Stable** if for every bounded input to the **system** there is a bounded output over the time interval. Mathematically it can be said that if input is integratable/summable over the entire interval of time the output is also integratable/summable for the system to be stable.

$\int y(t)dt < \infty \text{ for } \int x(t)dt < \infty$ for continuous time systems

$\sum y[n] < \infty \text{ for } \sum x[n] < \infty$ for discrete time systems

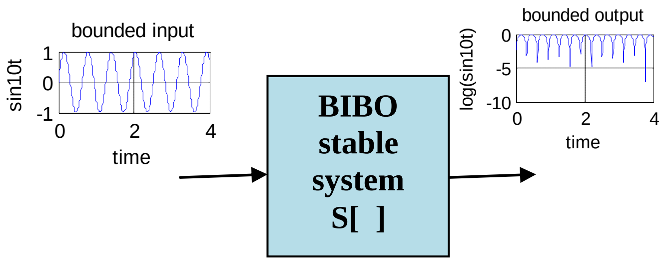


Fig. 12: Representation of Stable System

*Examples of stable systems*

- Consider a system with input x(t)=sin(10t) and output y(t)=log(x(t)). From the given diagram it can be seen that it is stable as bounded input gives bounded output.

*Program*

```
t = 0:0.01:4;
u = sin(10*t);
subplot(211);
plot(t,u);
xlabel('time');
ylabel('sin10t');
title('bounded input');
grid;
y = log(u);
subplot(212);
plot(t,y);
xlabel('time');
ylabel('log(sin10t)');
title('bounded output');
grid;
```

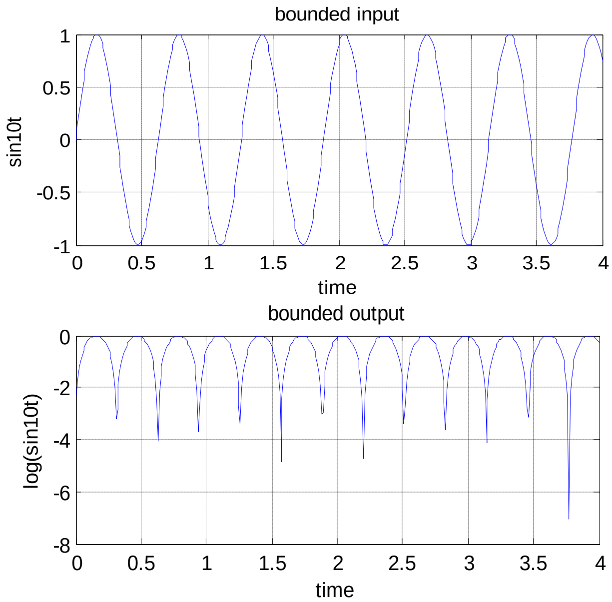


Fig. 13: Example BIBO stable system (a) shows input sin10t, (b) shows output log(sin10t)

- Consider a system with input x(t)=log(t) and output y(t)=sin(x(t)). From the given diagram it can be seen that it is stable as bounded input gives bounded output.

*Program*

```
t = 0:10000000;
u = log(t);
subplot(211);
plot(t,u);
xlabel('time');
ylabel('log10t');
title('bounded input');
grid;
y = sin(u);
subplot(212);
plot(t,y);
xlabel('time');
ylabel('sin(log10t)');
title('bounded output');
grid;
```

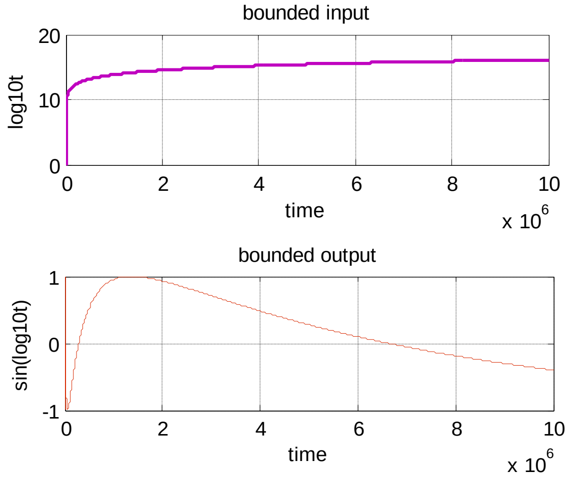


Fig. 14: Example BIBO stable system (a) shows input log10t, (b) shows output sin(log10t)

- Consider a system with input x(t)=t and output y(t)=exp(x(t)). From the given diagram it can be seen that it is unstable as unbounded input gives unbounded output.

*Program*

```
t = 0:100;
x=t;
subplot(211);
plot(t,x);
xlabel('time');
ylabel('x(t)');
title('unbounded input');
grid;
u = exp(t);
subplot(212);
plot(t,u);
xlabel('time');
ylabel('exp(t)');
title('unbounded output');
grid;
```

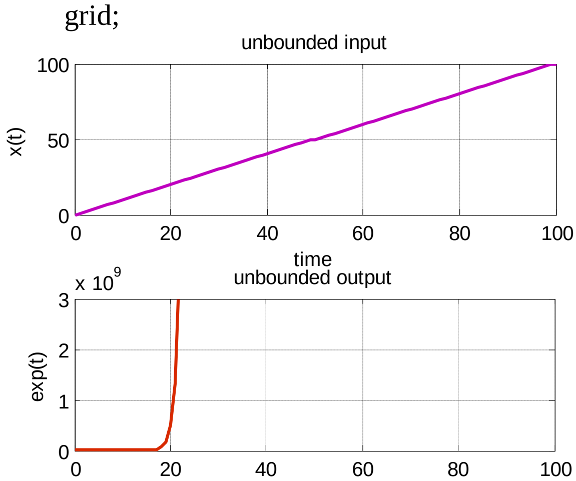


Fig. 15: example of ustable system (a) shows input x(t)=t, (b) shows output y(t)= exp(t)

## IV. CONCLUSION

The processes that are taking place in our environment in any field such as engineering, medical, finance, biological, or any such field can be classified into one of the categories mentioned above in this article. With the prior information about these systems any kind of signal operation can be done easily whether signal processing, signal transformation, etc…

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