Some properties of fuzzy Neutrosophic Bimatrices

[1]Dr. N. Elumalai, [2]Mr. R. Manikandan, [3]Ms. K.Kirubha

[1]Associate professor,[2]Assistant Professor, [3]Research Scholar [1,2,3] Department of Mathematics, A.V.C. College (Autonomous), Mannampandal, Tamilnadu

ABSTRACT

We study on some properties of fuzzy neutrosophic bimatrices are introduced and discussed an important results.

KEY WORD: *Bimatrix, fuzzy bimatrix, Neutrosophic fuzzy bimatrix.*

I. INTRODUCTION

Determined that bimatrices which are in fuzzy matrices. We discuss the above concepts of fuzzy bimatrices and it is analogous to that of fuzzy neutrosophic matrices. The sum and the product can also be applied on these matrices. The relationship between the fuzzy neutrosophic and neutrosophic bimatrices are discussed. The concept of fuzzy neutrosophic matrices [1].Properties of fuzzy matrices given in [5],[6]. In this paper, to define fuzzy bimatrices and also we discussed some properties on fuzzy neutrosophic bimatrices.

II. SOME OF DEFINITIONS AND PROPERTIES

Definition: 1[4]

A bimatrix A_B is defined as the union of two square array of numbers A_1 and A_2 arranged into rows and columns. It is written as follows $A_B = A_1 \cup A_2$

where $A_1 \neq A_2$ with

$$A_1 = \begin{bmatrix} a_{11}^1 & a_{12}^1 & \dots\dots & a_{1n}^1 \\ a_{21}^1 & a_{22}^1 & \dots\dots & a_{2n}^1 \\ \vdots & \vdots & & \vdots \\ a_{m1}^1 & a_{m2}^1 & \cdots & a_{mn}^1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} a_{11}^2 & a_{12}^2 & \dots\dots & a_{1n}^2 \\ a_{21}^2 & a_{22}^2 & \dots\dots & a_{2n}^2 \\ \vdots & \vdots & & \vdots \\ a_{m1}^2 & a_{m2}^2 & \cdots & a_{mn}^2 \end{bmatrix}$$

`∪` the notational convenience (symbol) only.

Definition:2

A fuzzy matrix A of order mxn is defined as $A=[<a_{ij}, a_{ij}\mu>]_{mxn}$ where $a_{ij}\mu$ is the membership value of the element a_{ij} in A. For simplicity, we write A as $A=[a_{ij}\mu]$.

Definition:3

If $a_{ij} \le rA(i) \wedge c_A(j)$ for all i, j then the FMFRC $A= [r_A(i)][c_A(j)]\left[a_{ij}\right]_{m\times n}$ is called g-FMFRC.

Complement of fuzzy matrix is defined. But for FMFRC two types of complement are defined.

The first type of complement is called c-complement and second type is called b-complement.

Definition: 4[2]

Let $A=[r_A(i)][c_A(j)]\left[a_{ij}\right]_{m\times n}$ be a g-FMFRC. The complement of the g-FMFRC A is denoted by $\bar{A}$ and it is defined as $\bar{A}=[\bar{r}_A(i)][\bar{c}_A(j)]\left[\bar{a}_{ij}\right]_{m\times n}$ where $\left[\bar{r}_A(i)\right]=r_A(i),\left[\bar{c}_A(j)\right]=c_A(j)$ and $\bar{A}=r_A(i)\wedge c_A(j)-a_{ij}$ for all i, j here also represents the ordinary subtraction.

Definition: 5

Let A= $[r_A(i)][c_A(j)][a_{ij}]_{m\times n}$ be a FMFRC.

Its complement is denoted by A^c and it is defined as

$A^c = [1-r_A(i)][1-c_A(j)][1-c_A(j)][1-a_{ij}]_{m\times n}$

.This complement is called c-complement.

Definition: 6

Let A=$[r_A(i)][c_A(j)][a_{ij}]_{m\times n}$ be a FMFRC. The density of A is denoted as D(A) and is defined as provided $\sum_{i,j} r_A \wedge c_A(j) \neq 0.$

Definition: 7

A FMFRC 'A' is called balanced if $D(A) \leq D(A)$ for all sub-FMFRC$_c$ S of A.

Now we define a particular type of balanced FMFRC.

Definition: 8

A FMFRC 'A' is called strictly balanced if $D(S) = D(A)$ for all sub-FMFRC$_c$ S of A.

Definition: 9

A matrix system such

$$\begin{pmatrix} a_{11} & a_{12} & \cdot & \cdot & \cdot & a_{1n} \\ a_{21} & a_{22} & \cdot & \cdot & \cdot & a_{2n} \\ \cdot & \cdot & & & & \cdot \\ \cdot & \cdot & & & & \cdot \\ \cdot & \cdot & & & & \cdot \\ a_{n1} & a_{n2} & \cdot & \cdot & \cdot & a_{nn} \end{pmatrix} \begin{pmatrix} x_{11} & x_{12} & \cdot & \cdot & \cdot & x_{1n} \\ a_{21} & a_{22} & \cdot & \cdot & \cdot & x_{2n} \\ \cdot & \cdot & & & & \cdot \\ \cdot & \cdot & & & & \cdot \\ \cdot & \cdot & & & & \cdot \\ x_{n1} & x_{n2} & \cdot & \cdot & \cdot & x_{nn} \end{pmatrix} = \begin{pmatrix} y_{11} & y_{12} & \cdot & \cdot & \cdot & y_{1n} \\ y_{21} & y_{22} & \cdot & \cdot & \cdot & y_{2n} \\ \cdot & \cdot & & & & \cdot \\ \cdot & \cdot & & & & \cdot \\ \cdot & \cdot & & & & \cdot \\ y_{n1} & y_{n2} & \cdot & \cdot & \cdot & y_{nn} \end{pmatrix}$$

Where $a_{ij}, 1 \leq i, j \leq n$ are real numbers the elements y_{ij} in the right hand matrix are fuzzy members and the unknown elements x_{ij} are ones is called a Fuzzy matrix equation system.

Using matrix notation, we've $AX = Y$

A fuzzy number matrix, X=($x_1 \ldots x_j \ldots x_n$) is called a solution of the fuzzy matrix system if $Ax_j = y_j$, $1 \leq j \leq n$.

Definition: 10[3]

Let N$_s$ ={[0,1]$\cup$ nI/n$\in(0,1]$}; we call the set N$_s$ to be the fuzzy neutrosophic set. Let N$_s$ be the fuzzy neutrosophic set. $M_{m\times n} = \{(a_{ij}) / a_{ij} \in N_s\}$ we call the matrices with entries from N$_s$ to be the fuzzy neutrososphic matrices.

Remarks :

Let N$_s$=$[0,1] \cup \{nI / n \in (0,1]\}$ be the set

$$P=\begin{pmatrix} 0 & 0.2I & I \\ I & 0.01I & 0 \\ 0.31I & 0.53I & 0.1 \end{pmatrix}$$

is a 3×3 fuzzy neutrosophic matrix.

Definition: 11

Let A be a neutrosophic fuzzy matrices, whose entries is of the form a+Ib (neutrosophic number), where a,b are the elements of [0,1] and I is an indeterminate such that I^n=I,n being a positive integer.

$$A = \begin{pmatrix} a_1 + Ib_1 & a_2 + Ib_2 \\ a_3 + Ib_3 & a_4 + Ib_4 \end{pmatrix}$$

Definition: 12

A bimatrix A_B is defined as the union of two rectangular or square array of numbers A_1 and A_2 arranged into rows and coloumns.

A_B=$A_1 \cup A_2$ where $A_1 \neq A_2$

'$\cup$' is just the notational convenience (symbol) only.

Definition: 13

Let A_B=$A_1 \cup A_2$ be a bimatrix. If both A_1 and A_2 are square matrices then A_B is called the square bimatrix.

If one of the matrices in the bimatrix A_B=$A_1 \cup A_2$ is square and other is rectangular or if both A_1 and A_2 are rectangular matrices say $m_1 \times n_1$ and $m_2 \times n_2$ with $m_1 \neq m_2$ or $n_1 \neq n_2$ then we say A_B is a mixed bimatrix

Definition: 14

Let $A=A_1 \cup A_2$ where A_1 and A_2 are two distinct fuzzy matrices with entries from the interval [0, 1]. Then $A=A_1 \cup A_2$ is called the fuzzy bimatrix.

Note:

1. If both A_1 and A_2 are fuzzy matrices we call A a fuzzy bimatrix.
2. If only of A_1 or A_2 is a fuzzy matrix then we call $A=A_1 \cup A_2$ as the semi fuzzy bimatrix. (It is clear all fuzzy matrices are trivially semi fuzzy matrices).

If both A_1 and A_2 are $m \times n$ fuzzy matrices yhen we call $A=A_1 \cup A_2$ a $m \times n$ fuzzy bimatrix or a rectangular fuzzy bimatrix.

Definition: 15

Let $A=A_1 \cup A_2$ where A_1 and A_2 are two distinct neutrosophic matrices with entries from a neutrosophic field. Then $A=A_1 \cup A_2$ is called the neutrosophic bimatrix.

Note:

(1) If both A_1 and A_2 are neutrosophic matrices we call A a neutrosophic bimatrix.
(2) If only one of A_1 or A_2 is a neutrosophic matrix and other is not a neutrosophic matrix then we all $A=A_1 \cup A_2$ as the semi neutrosophic bimatrix. (It is clear all neutrosophic bimatrices are trivially semi neutrosophic bimatrices).

It both A_1 and A_2 are $m \times n$ neutrosophic matrices then we call $A=A_1 \cup A_2$ a $m \times n$ neutrosophic bimatrix or a rectangular neutrosophic bimatrix.

Definition: 16

We know Z is the abelian group under addition.Z(I) denote the additive abelian group generated by the set Z and I,Z(I) is called the neutrosophic abelian group under'+'.

Definition: 17

Let G (I) be the neutrosophic group under addition. $P \subset G(I)$ be a proper subset of G(I). P is said to be the neutrosophic subgroup of G (I) if P itself is a neutrosophic group i.e., $P=\langle p_1 \cup I \rangle$ where P_1 is an additive subgroup of G.

Definition: 18

Let G (I) by an additive abelien neutrosophic group. K any field. If G(I) is a vector space over K then we call G(I) a neutrosophic vector space over K.

Definition: 19

Let $A_{FN}=A_1 \cup A_2$ where A_1 and A_2 are distinct integral fuzzy neutrosophic bimatrix. If bothe A_1 and A_2 are $m \times m$ distinct integral fuzzy neutrosophic matrix then $A_{FN}=A_1 \cup A_2$ is called the square integral fuzzy neutrosophic bimatrix.

Example:

$$A_{FN} = \begin{bmatrix} 0 & I & .3 \\ 2I & .4 & 1 \\ 0 & .3 & -.6 \end{bmatrix} \begin{bmatrix} 1 & I & 0 \\ I & 1 & .8 \\ .6 & 1 & .7I \end{bmatrix}$$

Definition: 20

Let G (I) be a neutrosophic additive abelian group. K (I) be a neutrosophic field. If G(I) is a vector space over K(I) then we call G(I) the strong neutrosophic vector space.

ADDITION OPERATION OF TWO NEUTROSOPHIC FUZZY BIMATRICES:

$$A_{FN} + B_{FN} = [Cij]_{FN} \text{ Where}$$

$$C_{11} = \max(a_1, c_1) + I \max(b_1, d_1)$$
$$C_{12} = \max(a_2, c_2) + I \max(b_2, d_2)$$
$$C_{21} = \max(a_3, c_3) + I \max(b_3, d_3)$$
$$C_{22} = \max(a_4, c_4) + I \max(b_4, d_4)$$

Properties:

(i) $A_{FN}+B_{FN}=B_{FN}+A_{FN}$
(ii) $(A+B)_{FN}+C_{FN}=A_{FN}+(B+C)_{FN}$

Multiplication Operation of Neutrosophic Fuzzy Bimatrices:

Let us consider two neutrosophic fuzzy matrices as $A_{FN} = [a_{ij} + Ib_{ij}]$ and $B_{FN}=[c_{ij}+Id_{ij}]$. Then we shall define the multiplication of these two nuetrosophic fuzzy matrices as

$A_{FN}B_{FN}=[\max\min(a_{ij},c_{ij})+I\max\min(b_{ij},d_{ij})]_{FN}$

$(AB)_{FN}=[D_{ij}]_{FN}$

Where,

$D_{11}=[\max\min\{(a_1,c_1),(a_2,c_3)\}+I\max\min\{(b_1,d_1),(b_2,d_3)\}]$

$D_{12}=[\max\min\{(a_1,c_2),(a_2,c_4)\}+I\max\min\{(b_1,d_2),(b_2,d_4)\}]$

$D_{21}=[\max\min\{(a_3,c_1),(a_4,c_3)\}+I\max\min\{(b_3,d_1),(b_4,d_3)\}]$

$D_{22}=[\max\min\{(a_3,c_2),(a_4,c_4)\}+I\max\min\{(b_3,d_2),(b_4,d_4)\}]$

Properties:

(i) $A_{FN}B_{FN} \neq B_{FN}A_{FN}$

(ii) $A_{FN}(B_{FN}+C_{FN})=AB+AC$

Examples:

(i) $A_{FN}+B_{FN}=B_{FN}+A_{FN}$

$$A_1 = \begin{bmatrix} 0 & I & .3 \\ .2I & .4 & 1 \\ 0 & .3 & -.6 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 1 & I & 0 \\ I & 1 & .8 \\ .6 & 1 & .7I \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 0 & I & .2 \\ .2I & 4 & .1 \\ 0 & 3 & -.6 \end{bmatrix} \quad B_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$A_{FN}+B_{FN}=B_{FN}+A_{FN}$

$$(A_1 \cup A_2) + (B_1 \cup B_2) = (B_1 \cup B_2) + (A_1 \cup A_2)$$
$$(A_1 + B_1) \cup (A_2 + B_2) = (B_1 + A_1) \cup (B_2 + A_2)$$

$$\begin{bmatrix} 0 & I & .3 \\ .2I & 4 & 1 \\ 0 & 3 & -.6 \end{bmatrix} \cup \begin{bmatrix} 1 & I & 0 \\ I & 1 & .8 \\ .6 & I & .7I \end{bmatrix}$$

$$\begin{bmatrix} 0 & I & .3 \\ .2I & .4 & 1 \\ 0 & 3 & -.6 \end{bmatrix} \cup \begin{bmatrix} 1 & I & 0 \\ I & 1 & .8 \\ .6 & I & .7I \end{bmatrix}$$

$$(A_1 + B_1) \cup (A_2 + B_2) = (B_1 + A_1) \cup (B_2 + A_2)$$

(*ii*) (A+B)+C=A+(B+C)

$$(A_{FN} + B_{FN}) + C_{FN} = A_{FN} + (B_{FN} + C_{FN})$$
$$[(A_1 \cup A_2) + (B_1 \cup B_2)] + (C_1 \cup C_2) = (A_1 \cup A_2)[(B_1 \cup B_2) + (C_1 \cup C_2)]$$
$$[(A_1 + B_1) \cup (A_2 + B_2)] + (C_1 \cup C_2) = (A_1 \cup A_2) + [(B_1 + C_1) \cup (B_2 + C_2)]$$
$$(A_1 + B_1 + C_1) \cup (A_2 + B_2 + C_2) = (A_1 + B_1 + C_1) \cup (A_2 + B_2 + C_2)$$

$$A_1 = \begin{bmatrix} 0 & I & 0 \\ 1 & 2 & -1 \\ 3 & 2 & I \end{bmatrix}; \quad A_2 = \begin{bmatrix} 2 & I & 0 \\ I & 0 & I \\ 1 & 1 & 2 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 2 & 0 & I \\ 4 & I & 1 \\ 1 & 1 & 0 \end{bmatrix};$$

$$B_2 = \begin{bmatrix} 3 & 0 & I \\ 0 & I & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$C_1=\begin{bmatrix}0 & I & 0\\ 2I & .4 & 1\\ 0 & .3 & 0\end{bmatrix};$$

$$C_2=\begin{bmatrix}2 & 0 & I\\ 0 & 0 & 0\\ 2 & 3 & 0\end{bmatrix}$$

$(A_1+B_1+C_1)\cup(A_2+B_2+C_2)$

$$=\begin{bmatrix}2 & I & I\\ 4 & 2 & 1\\ 3 & 2 & I\end{bmatrix}\cup\begin{bmatrix}3 & I & I\\ I & I & I\\ 2 & 3 & 2\end{bmatrix}$$

(A+B)+C=A+(B+C)

(iii) A(B+C)=AB+AC

$$A(B+B)=A_{FN}(B_{FN}+C_{FN})$$

$$=A_1\cup A_2(B_1\cup B_2+C_1\cup C_2)$$
$$=A_1\cup A_2(B_1+C_1\cup B_2+C_2)$$
$$=A_1(B_1+C_1)\cup A_2(B_2+C_2)$$
$$A_{FN}(B+C)_{FN}=A_{FN}B_{FN}+A_{FN}C_{FN}$$

$$A_1=\begin{bmatrix}1+I2 & 5+I3\\ 6+I8 & 2+I4\end{bmatrix}$$
$$A_2=\begin{bmatrix}6+I2 & 5+I1\\ 8+I3 & 2+I5\end{bmatrix}$$

$$B_1=\begin{bmatrix}3+I2 & 6+I5\\ 5+I1 & 7+I4\end{bmatrix}$$
$$B_2=\begin{bmatrix}2+I4 & 7+I8\\ 9+I2 & 6+I7\end{bmatrix}$$

$$C_2=\begin{bmatrix}5+I3 & 2+I4\\ 1+I7 & 6+I1\end{bmatrix}$$
$$C_2=\begin{bmatrix}1+I6 & 4+I5\\ 2+I1 & 7+I3\end{bmatrix}$$

$$A_1(B_1+C_1)\cup A_2(B_2+C_2)$$
$$(A_1B_1+A_1C_1)\cup(A_2B_2+A_2C_2)$$

A(B+C)=AB+AC

(iv) $(A')'_{FN}=A_{FN}$

$$(A_1'\cup A_2')'=A_1\cup A_2$$

$$A_1=\begin{bmatrix}2 & I & 1\\ 3 & 5 & I\\ 0 & 1 & 1\end{bmatrix}$$

$$A_2=\begin{bmatrix}1 & 0 & I\\ 5 & 2 & 1\\ 4 & I & 6\end{bmatrix}$$

$$\because (A')'=A$$

$$(A_1'\cup A_2')'=A_1\cup A_2$$

(v) $(A+B)_{FN}'=A_{FN}'+B_{FN}'$

$$[(A_1\cup A_2)+(B_1\cup B_2)]'=(A_1'\cup A_2')+(B_1'\cup B_2')$$
$$[A_1+B_1)\cup(A_2+B_2)]'=(A_1'+B_1')\cup(A_2'+B_2')$$
$$(A_1'+B_1')\cup(A_2'+B_2')=(A_1'+B_1')\cup(A_2'+B_2')$$

$$A_1'=\begin{bmatrix}7 & 5 & I\\ 0 & 6 & I\\ 2 & 4 & 3\end{bmatrix}\quad A_2'=\begin{bmatrix}2 & 0 & 1\\ I & I & 3\\ 5 & 7 & 4\end{bmatrix}$$

$$B_1'=\begin{bmatrix}3 & 2 & I\\ 5 & 0 & 1\\ 2 & I & I\end{bmatrix}\quad B_2'=\begin{bmatrix}6 & 0 & 1\\ I & 4 & 3\\ I & 0 & I\end{bmatrix}$$

$$(A_1'+B_1')\cup(A_2'+B_2')=\begin{bmatrix}7 & 5 & I\\ 5 & 6 & I\\ 2 & 4 & 3\end{bmatrix}\cup\begin{bmatrix}6 & 0 & 1\\ I & 4 & 3\\ 5 & 7 & 4\end{bmatrix}$$
$$(A_1'+B_1')\cup(A_2'+B_2')=(A_1'+B_1')\cup(A_2'+B_2')$$

$$(A+B)'=A'+B'$$

$$(vi)(A.B)_{FN}' = B_{FN}'.A_{FN}'$$

$$[(A_1 \cup A_2)(B_1 \cup B_2)] = (B_1' \cup B_2')(A_1' \cup A_2')$$

$$(A_1B_1 \cup A_2B_2)' = B_1'A_1' \cup B_2'A_2'$$
$$A_1'B_1' \cup A_2'B_2' = B_1'A_1' \cup B_2'A_2'$$

$$A_1' = \begin{bmatrix} 0 & 6 & I \\ 2 & 1 & 5 \\ 4 & 0 & I \end{bmatrix}$$
$$A_2' = \begin{bmatrix} 7 & I & I \\ 6 & 0 & 1 \\ 5 & 2 & I \end{bmatrix}$$

$$B_1' = \begin{bmatrix} 5 & 2 & 0 \\ I & 1 & 6 \\ I & 0 & I \end{bmatrix} \quad B_2' = \begin{bmatrix} 0 & 0 & I \\ 1 & 6 & 5 \\ 2 & 4 & I \end{bmatrix}$$

$$A_1'B_1' \cup A_2'B_2' \neq B_1'A_1' \cup B_2'A_2'$$

$(vii) A.A_{FN}'$ and $A'.A_{FN}$ are both symmetric

$(A_1 \cup A_2)(A_1' \cup A_2')$ and
$(A_1' \cup A_2')(A_1 \cup A_2)$

$A_1A_1' \cup A_2A_2'$ and $A_1'A_1 \cup A_2'A_2$

$$A_1 = \begin{bmatrix} 0 & I & 1 \\ 2 & 4 & 6 \\ 7 & 5 & I \end{bmatrix} \quad A_1' = \begin{bmatrix} 7 & 5 & I \\ 5 & 6 & I \\ 2 & I & 3 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 6 & 0 & 1 \\ I & 4 & 3 \\ I & 5 & 7 \end{bmatrix}$$
$$A_2' = \begin{bmatrix} 0 & 1 & 5 \\ 7 & 6 & I \\ I & I & 0 \end{bmatrix}$$

$$A_1A_1' \cup A_2A_2' = \begin{bmatrix} I & I & I \\ 4 & 4 & 3 \\ 7 & 5 & 1 \end{bmatrix} \cup \begin{bmatrix} I & I & 5 \\ 4 & 4 & I \\ 5 & 5 & I \end{bmatrix}$$
$$A_1'A_1 \cup A_2'A_2 = \begin{bmatrix} 2 & 4 & 5 \\ 2 & 4 & 6 \\ 3 & 3 & I \end{bmatrix} \cup \begin{bmatrix} I & 5 & 5 \\ 6 & 4 & 3 \\ I & I & I \end{bmatrix}$$

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