Effect of Time Periodic Body Force on the Onset of Rayleigh Bénard Electro Convection in a Weakly Conducting Couple Stress Fluid in a Saturated Porous Medium

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ABSTRACT

The effect of the electric field along with time periodic body force (Gravity modulation) is studied with couple stress fluid in porous medium which is a weak conductor of electricity. Linear analysis of the problem is discussed. The Rayleigh number is obtained by using Perturbation procedure given by Venezian. The Rayleigh number with modulation is obtained in terms of Critical Rayleigh number and some non-dimensional parameters related to the problem.

Keywords *- Electric field, Gravity modulation, Rayleigh-Bénard Convection, weakly conducting couple stress fluid.*

1. INTRODUCTION

The theory of Rayleigh-Bénard convection in a layer of liquid of infinite length has significant uses in various science and technology problems like extraction of oil from porous medium etc.

Gravitational force is not constant all across the universe in the space. The space laboratories which are constructed in space crafts are under constant movement due to the rotation and revolutions of spacecraft. So, acceleration due to gravity in these space laboratories will be varying all the time. This variating gravity is called as Gravity modulation (g-jitter). This variational gravity can either delay or advance the heat transfer which in turn affect the convection in the system. The effect of Gravity modulation can affect the stability of the system under consideration may be either by increasing or decreasing the rate of convection.

Benjamin and Ursell [1] first studied the effect of gravity modulation using the Venezian [2] approach. Gresho and sani [3], Malashetty and Begum [4], S. Pranesh and Sameena Tarannum [5], P. G. Siddeshwar and S. Pranesh [6], wheeler *et al.* [7] studied the effects of g-jitters in different fluids and conditions.

Studies on non-Newtonian fluids is given more attention due to its wide range of applications in engineering and atmospheric science. One such non-Newtonian fluids is Couple stress fluid which has unique features like polar effects and considerably large viscosity. Considering the couple stress with Classical Cauchy stress has paved way for the development of couple stress fluid theory. One of the prominent and earliest couple stress theory is Stokes [8] couple stress theory. It gives the generalization of the Cauchy's theory which considers the presence of body couple and couple stresses. Pandey and Chaube [9], Sharma and Monika [10], R. C. Sharma and S. Sharma [11], Sunil *et al.* [12] and many more studied the heat transfer in couple-stress fluid under various conditions.

In last century applied fluid mechanics in engineering was studied with absence of electromagnetic fields. Now-a-days, the study effects of electric and magnetic fields on fluids drew lot of attention in fields of chemical engineering, nuclear physics, medicine and etc. Rudraiah *et al.* [13], Siddheshwar and Abraham [14] and Takashima and Gosh [15] studied the effects of electric field on convection.

The main aim of this paper is to study the effects of electric field and time periodic body force in the process of convection in a couple stress fluid which is a weak conductor of electric current with a saturated porous layer.

2. MATHEMATICAL FORMULATION

Consider two parallel plates of infinite length and separated by the distance d (Figure 1). A layer of infinite length of couple stress fluid in porous media is taken. Gravity is acting vertically downwards and applied electric field is vertically upwards. Lower plate is assumed to be hotter than the upper plate.

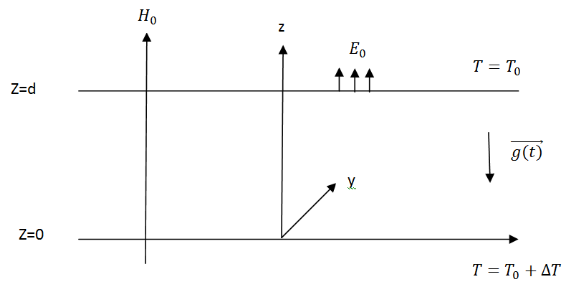

Fig 1: Physical Configuration of the problem under consideration

The Governing Equations are:

Continuity Equation:

$$\nabla.\vec{q} = 0 \tag{1}$$

Conservation of Linear Momentum:

$$\rho_0\left[\frac{1}{\varepsilon}\frac{\partial \vec{q}}{\partial t} + \frac{1}{\varepsilon^2}(\vec{q}.\nabla)\vec{q}\right] = -\nabla p - \rho g(t) + \frac{\mu'}{K}\nabla^2\vec{q} - \frac{\mu_e}{K}\vec{q} + \vec{J}\times\vec{B} + (\vec{P}.\nabla)\vec{E} \tag{2}$$

$$g(t) = g_0(1+\delta \mathrm{Cos}(\gamma t))\hat{k} \tag{3}$$

Conservation of Energy:

$$\gamma'\frac{\partial T}{\partial t} + (\vec{q}.\nabla)T = \chi\nabla^2 T \tag{4}$$

Equation of State:

$$\rho = \rho_0\left[1-\alpha(T-T_0)\right] \tag{5}$$

Constitutive Equation :

$$\vec{J} = \sigma(\vec{q}\times\vec{B}) \tag{6}$$

$$\vec{B} = \mu_m\vec{H} \tag{7}$$

Faraday's Law:

$$\nabla\times\vec{E} = 0 \text{ and } \vec{E} = -\nabla\phi \tag{8}$$

Equation of Polarization Fluid:

$$\nabla.(\varepsilon_0\vec{E}+\vec{P}) = 0 \text{ and } \vec{P} = \varepsilon_0(\varepsilon_r - 1)\vec{E} \tag{9}$$

Equation of state of Dielectric Constant:

$$\varepsilon_r = (1+\chi_e) - e(T-T_0) \tag{10}$$

$\vec{q}$ is the fluid velocity, ρ_0 is the density at reference temperature T, ε is the porosity, p is the Hydrodynamic pressure, ρ is the density, g(t) is the gravitational force, μ' is the couple stress viscosity, K is the permeability of porous medium, μ_e is the effective viscosity, $\vec{J}$ is the current density, $\vec{B}$ is the magnetic induction vector, $\vec{P}$ is the Dielectric polarization, $\vec{E}$ is the Electric field, g_0 is the mean gravity, δ is the small amplitude of g-jitters, γ is the frequency of modulation, t is the time, γ' is the ratio of heat capacity, T is the Temperature, χ is the thermal conductivity, α is the co-efficient of thermal expansion, σ is the magnetic conductivity, μ_m is the magnetic permeability, ε_r $\vec{H}$ is the magnetic field, ϕ is the electric potential, ε_0 is the electric χ_e permittivity of free space, is the Dielectric constant, is the Electric susceptibility, e is the Dielectric permittivity.

Due to the weak conductance of electric field by the fluid, the Lorentz force $\vec{J}\times B$ by considering the equations (6) and (7) can be written as

$$\vec{J}\times\vec{B} = -\mu_m{}^2\sigma H_0{}^2\vec{q} \tag{11}$$

Where H_0 is the applied magnetic field.

1.1 BASIC STATE

The basic state is assumed to be at rest and we consider the solution of the form given by

$$\begin{gathered}\vec{q}_b = 0,\ p = p_b(z),\ \rho = \rho_b(z),\ T = T_b(z),\\ \vec{E} = E_b(z),\ \vec{P} = P_b(z),\ \varepsilon_r = \varepsilon_{r_b}(z),\ \phi = \phi_b(z)\end{gathered} \tag{12}$$

where basic state is denoted by subscript b.

Substituting (12) in (1)-(11) gives the basic state equations as

$$-\nabla p_b - \rho_b g(t) + (\vec{P}_b.\nabla)\vec{E}_b = 0 \tag{13}$$

$$T_b = \frac{-\Delta T}{d} z + T_1 \tag{14}$$

$$\rho_b = \rho_0 \left[1 - \alpha(T_b - T_0)\right] \tag{15}$$

$$\varepsilon_{r_b} = (1+\chi_e) - e(T_b - T_0) \tag{16}$$

$$\vec{E}_b = \left[\frac{E_0(1+\chi_e)}{(1+\chi_e) - e\dfrac{\Delta T}{d} z} \right] \hat{k} \tag{17}$$

$$\vec{P}_b = \varepsilon_0 (1+\chi_e) \left[1 - \frac{1}{(1+\chi_e) - e\dfrac{\Delta T}{d} z} \right] \hat{k} \tag{18}$$

2.2 LINEAR STABILITY ANALYSIS

The infinitesimal disturbances is introduced in to the system in order to analyze the stability, which is of the form

$$\begin{aligned} &\vec{q} = \vec{q}_b + \vec{q}', \; p = p_b + p', \\ &\rho = \rho_b + \rho', \; T = T_b + T', \\ &\vec{E} = E_b + \vec{E}', \; \vec{P} = P_b + \vec{P}', \\ &\varepsilon_r = \varepsilon_{r_b} + \varepsilon_r', \; \phi = \phi_b + \phi', \end{aligned} \tag{19}$$

Where perturbations are denoted by primes and assumed to be very small.

Substituting (19) in (1)-(11) and using Equations (13)-(18) and Linearizing the resulting equations. We get Perturbed state Equations as

$$\nabla.\vec{q}' = 0 \tag{20}$$

$$\begin{aligned} \frac{\rho_0}{\varepsilon}\frac{\partial \vec{q}'}{\partial t} = &-\nabla p' - \rho' g_0 (1 + \delta \mathrm{Cos}(\gamma t))\hat{k} + \\ &\frac{\mu'}{K}\nabla^2 \vec{q}' - \frac{\mu_e}{K}\vec{q}' - \mu_m^{\,2}\sigma H_0^{\,2}\vec{q}' + \\ &(\vec{P}'.\nabla)\vec{E}' + \vec{P}_b.\nabla\vec{E}' \end{aligned} \tag{21}$$

$$\gamma' \frac{\partial T'}{\partial t} = \frac{\Delta T}{d} w' + \chi \nabla^2 T' \tag{22}$$

$$\rho' = -\rho_0 \alpha T' \tag{23}$$

$$\varepsilon_r' = (1+\chi_e) - eT' \tag{24}$$

$$\vec{E}' = -\nabla \phi' \tag{25}$$

$$\nabla^2 \phi' (1+\chi_e) - eE_0 \frac{\partial T'}{\partial t} = 0 \tag{26}$$

Operating Curl twice on equation (21) in order to eliminate pressure and using Equation (23), we get

$$\begin{aligned} \frac{-\rho_0}{\varepsilon}\frac{\partial}{\partial t}(\nabla^2 \vec{w}') = &-\rho_0 \alpha g_0 (1 + \\ &\delta \mathrm{Cos}(\gamma t))\hat{k}(\nabla_1^{\,2} T') \\ &-\frac{\mu'}{K}(\nabla^4 \vec{w}') + \frac{\mu_e}{K}(\nabla^2 \vec{w}') + \\ &\mu_m^{\,2}\sigma H_0^{\,2}(\nabla^2 \vec{w}') + \\ &\frac{\varepsilon_0 e E_0 \Delta T}{d}\left(\frac{\partial}{\partial z}(\nabla_1^{\,2}\phi')\right) - \\ &\frac{\varepsilon_0 e E_0 \Delta T}{d(1+\chi_e)}\left(\frac{\partial}{\partial z}(\nabla_1^{\,2}\phi')\right) \\ &-\frac{\varepsilon_0 \left(eE_0\right)^2 \Delta T}{d(1+\chi_e)}\left(\nabla_1^{\,2} T'\right) \end{aligned} \tag{27}$$

Non-Dimensionalising the Equations (22), (26) and (27) using the following scales

$$(x', y', z') = (x^*, y^*, z^*)d, \; t' = t^* \frac{\varepsilon d^2}{\chi},$$
$$w' = w^* \frac{\chi}{d}, \nabla' = \nabla^* \frac{1}{d}, \phi' = \frac{eE_0 d \Delta T}{(1+\chi_e)}\phi^*$$

We obtain,

$$\begin{aligned} \frac{1}{\mathrm{Pr}}\frac{\partial}{\partial t}(\nabla^2 \vec{w}) = &R(1 + \delta \mathrm{Cos}(\Omega t))\nabla_1^{\,2} T + \\ &C\nabla^4 \vec{w} - \frac{1}{Da}\nabla^2 \vec{w} - M^2 \nabla^2 \vec{w} - \\ &L\frac{\partial}{\partial z}(\nabla_1^{\,2}\phi) + L\nabla_1^{\,2} T \end{aligned} \tag{28}$$

$$\left(M_1 \frac{\partial}{\partial t} - \nabla^2\right) T = \vec{w} \tag{29}$$

$$\nabla^2\phi = \frac{\partial T}{\partial z} \quad (30)$$

Asterisks have been dropped for simplicity and the non-dimensional parameters are $\Pr,\ R,\ \Omega,\ C,\ Da,\ M^2,\ L,\ M_1$ which are defined as

$\Pr = \frac{\rho_0\chi}{\varepsilon^2\mu_e}$ Prandtl Number

$R = \frac{\rho_0\alpha g_0\Delta T d^3}{\chi\mu_e}$ Rayleigh Number

$\Omega = \frac{\gamma\varepsilon d^2}{\chi}$, Non-Dimensional Frequency of Modulation

$C = \frac{\mu'}{K\mu_e}$ Couple Stress Parameter

$Da = \frac{K}{d^2}$ Darcy Number

$M^2 = \frac{\mu_m{}^2\sigma H_0{}^2 d^2}{\mu_e}$ Hartmann Number

$L = \frac{\varepsilon_0\left(eE_0\Delta T d\right)^2}{\mu_e\chi(1+\chi_e)}$ Electric Rayleigh Number

$M_1 = \frac{\gamma'}{\varepsilon}$ Porous Parameter

Equations (28), (29) and (30) are solved under the boundary conditions

$$w = D^2 w = T = \phi = 0 \text{ at } z=0,1$$

Eliminating w and ϕ from equation (28) using equations (29) and (30)

$$\left(\frac{1}{\Pr}\frac{\partial}{\partial t}\nabla^4 - C\nabla^6 + \frac{1}{Da}\nabla^4 + M^2\nabla^4\right)\left(M_1\frac{\partial}{\partial t} - \nabla^2\right)T = \left(R(1+\delta\,\mathrm{Cos}(\Omega t))\nabla^2 + L\frac{\partial^2}{\partial z^2} - L\nabla^2\right)\nabla_1{}^2 T \quad (31)$$

By solving equations (28) to (30), we obtain the temperature boundary conditions for equation (31) which is of the form

$$T = \frac{\partial^2 T}{\partial z^2} = \frac{\partial^4 T}{\partial z^4} = \frac{\partial^6 T}{\partial z^6} = 0 \text{ at } z=0,1 \quad (32)$$

2.3 PERTURBATION PROCEDURE

Let (R, T) be the Eigen value and Eigen functions of Equation (31) which is of the form

$$R = R_0 + \delta R_1 + \delta^2 R_2 + \ldots$$
$$T = T_0 + \delta T_1 + \delta^2 T_2 + \ldots \quad (33)$$

The expansion of (33) is substituted into (31) and comparing the like powers of δ^0, δ^1 and δ^2. We get

$$L_1 T_0 = 0 \quad (34)$$

$$L_1 T_1 = \nabla^2\nabla_1{}^2(R_0 f)T_0 \quad (35)$$

$$L_1 T_2 = \nabla^2\nabla_1{}^2(R_2 T_0 + R_0 f T_1) \quad (36)$$

Where

$$f = \mathrm{Cos}\,\Omega t$$

$$L_1 = \left(\frac{1}{\Pr}\frac{\partial}{\partial t}\nabla^4 - C\nabla^6 + \frac{1}{Da}\nabla^4 + M^2\nabla^4\right)\left(M_1\frac{\partial}{\partial t} - \nabla^2\right) + \frac{\partial^2}{\partial z^2}\nabla_1{}^2 - L\nabla^2\nabla_1{}^2 - \nabla^2\nabla_1{}^2 R_0 \quad (37)$$

2.3.1 SOLUTION TO THE ZEROTH ORDER PROBLEM

It involves solving the electro convection problem of weakly conducting couple stress fluid without Gravity Modulation. The trial solution for Equation (34) is of the form

$$T_0 = \mathrm{Sin}(\pi z)e^{i(lx+my)} \quad (38)$$

Eigen value with respect to lowest mode of convection is given by

$$R_o = \frac{Ck^8 + Da^{-1}k^6 + M^2k^6 - Lk^2a^2 + L\pi^2a^2}{k^2a^2} \quad (39)$$

Where

$$k^2 = \pi^2 + a^2$$

2.3.2 SOLUTION TO THE FIRST ORDER PROBLEM

Using Equation (38) on Equation (35), we get

$$L_1 T_1 = R_1 k^2 a^2 \operatorname{Sin}(\pi z) \quad (40)$$

In the above equation, the time independent part must be orthogonal to $\sin(\pi z)$ to have solution.

Since f is time, dependent sinusoidal wave. We take R_1=0 which implies that $R_1 = R_3 = \dots = 0$ in equation (33).

$$L_1 T_1 = R_0 k^2 a^2 f \operatorname{Sin}(\pi z) \quad (41)$$

Using equation (37), we get that

$$L_1\left[\operatorname{Sin}(\pi z) e^{i(lx+my-\Omega t)}\right] = L_1(\Omega)\operatorname{Sin}(\pi z) e^{i(lx+my-\Omega t)} \quad (42)$$

where

$$L_1(\Omega) = Y_1 + iY_2$$

$$Y_1 = Ck^8 + Da^{-1}k^6 + M^2 k^6 - \frac{M_1 k^4 \Omega^2}{\Pr} - Lk^2 a^2 + La^2 \pi^2 - R_0 k^2 a^2$$

$$Y_2 = \frac{-k^6 \Omega}{\Pr} - Da^{-1} M_1 k^4 \Omega - CM_1 \Omega k^6 - M^2 M_1 k^4 \Omega$$

The particular solution of equation (40) is

$$T_1 = \left\{ \frac{R_0 k^2 a^2}{|L(\Omega)|^2} \quad (\mathrm{Y_1 Cos\Omega t + Y_2 Sin\Omega t}) \right\} \operatorname{Sin}(\pi z) \quad (43)$$

We consider that T_0 is orthogonal to all other T_n's

The equation of T_2 is

$$L_1 T_2 = k^2 a^2 R_2 T_0 + k^2 a^2 R_0 T_1 f \quad (44)$$

We cannot not solve equation (44) due its high non-linearity, but we will determine R_2 from it. For the solution to exist for equation (44), the steady part of its R. H. S. should be orthogonal to $\operatorname{Sin}(\pi z)$. Taking time average

$$R_2 = -2R_0 \int_0^1 \overline{fT_1} \operatorname{Sin}(\pi z) dz \quad (45)$$

And finally

$$R_2 = -\frac{R_0 k^2 a^2 Y_1}{2|L_1(\Omega)|^2} \quad (46)$$

3. RESULTS AND DISCUSSIONS

The effect of the electric field is studied in couple stress fluid which is a weak conductor of electricity in porous medium for small amplitudes of time periodic body force (Gravity modulation). Importance is given for when the convection sets in. Venezian method is used in solving the considered modulation problem.

We assume that the problem is valid only for small amplitudes of frequency of gravity modulation. If the Non-dimensional frequency of modulation is high, the buoyancy force becomes more prominent and the problem becomes reduces to that of no modulation case.

Fig (2) is the graph of Correction Rayleigh number R_{2c} vs Non-dimensional frequency of modulation Ω along with variation of values of couple stress parameter C. We can conclude that as C value increase the value of R_{2c} is also increasing which stabilizes the system by delaying the onset of convection. C is the ratio of viscosity of the suspended particles to viscosity of the fluid under consideration. It indicates the extent of body couple and couple stress applied by the suspended particles onto the fluid. So more the suspended particles more the viscous force and more energy is required for convection to begin.

Fig (3) is the graph of Correction Rayleigh number R_{2c} vs Non-dimensional frequency of modulation Ω along with variation of values of Hartmann number M 2. We can see that as M 2 increases it will increase the value of R_{2c}. M 2 is the representation of strength of magnetic field in the system. Magnetic field is directly proportional to the viscous force of the fluid. Magnetic lines tries to align the particles according to the polarity. So the energy provided to the system first nullify the effect of magnetic field in order to introduce disturbances into the system. So M 2 stabilizes the system by delaying the onset of convection.

Fig (4) is the graph of Correction Rayleigh number R_{2c} vs Non-dimensional frequency of modulation Ω

along with variation of values of Inverse Darcy number Da^{-1}. From the graph we can conclude that R_{2c} is directly proportion to Inverse Darcy number. Increase in Da leads to the decrease in R_{2c} which decreases the permeability of the porous medium which in turn retards the fluid flow and thus destabilizing the system.

Fig (5) is the graph of Correction Rayleigh number R_{2c} vs Non-dimensional frequency of modulation Ω along with variation of values of Porous parameter M_1. As M_1 decreases, R_{2c} increases and thus destabilizing the system.

Fig (6) is the graph of Correction Rayleigh number R_{2c} vs Non-dimensional frequency of modulation Ω along with variation of values of Prandtl number Pr. Increase in Pr value, increases R_{2c} value. Pr affects only R_{2c} not R_0 as it is absent in the expression for R_0.

Fig (7) is the graph of Correction Rayleigh number R_{2c} vs Non-dimensional frequency of modulation Ω along with variation of values of electric Rayleigh number L. Increase in L decreases the value of R_{2c} which advances the convection which destabilize the system. L represents the strength the concentration of applied electric field. As we increase L, the system receives extra energy along with heating. So, the convection sets in faster as dissipative force becomes lesser and lesser.

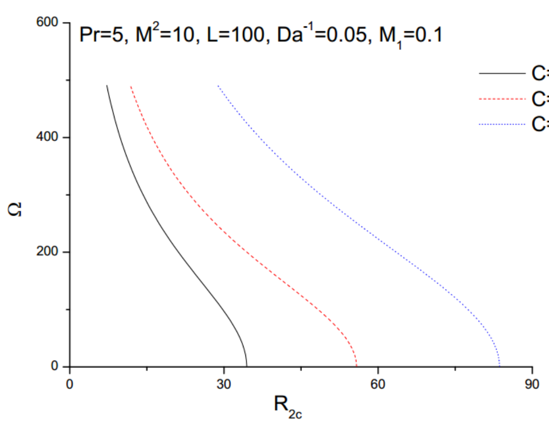

Fig 2: The graph of correction Rayleigh number R_{2c} vs Non-dimensional frequency of modulation Ω with variation in the values of Couple stress parameter C.

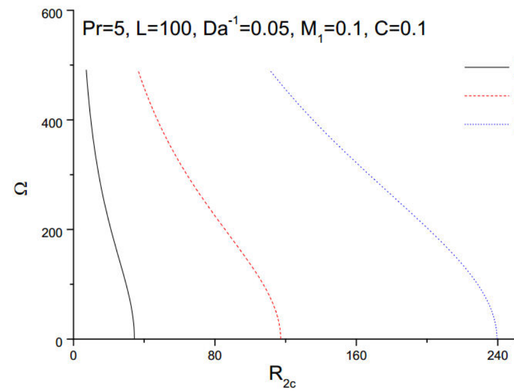

Fig 3: The graph of correction Rayleigh number R_{2c} vs Non-dimensional frequency of modulation Ω with variation in the values of Hartmann number M^2.

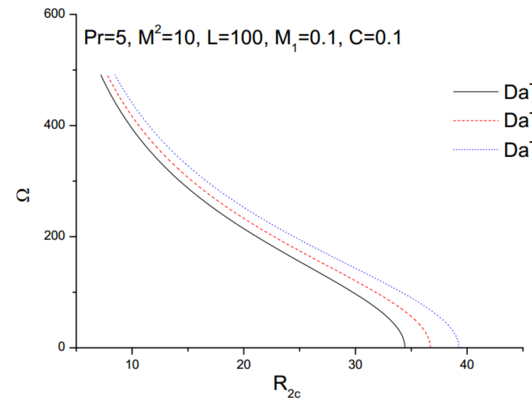

Fig 4: The graph of correction Rayleigh number R_{2c} vs Non-dimensional frequency of modulation Ω with variation in the values of Inverse Darcy Number Da^{-1}.

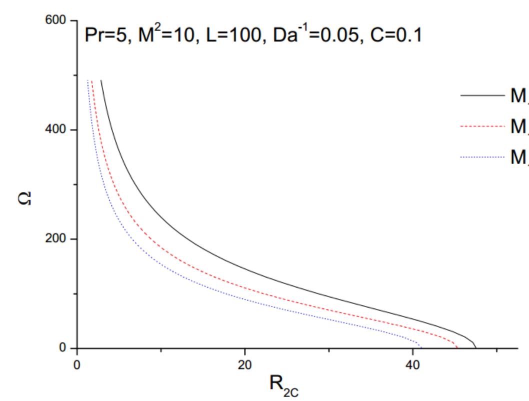

Fig 5: The graph of correction Rayleigh number R_{2c} vs Non-dimensional frequency of modulation Ω with variation in the values of Porous Parameter M_1.

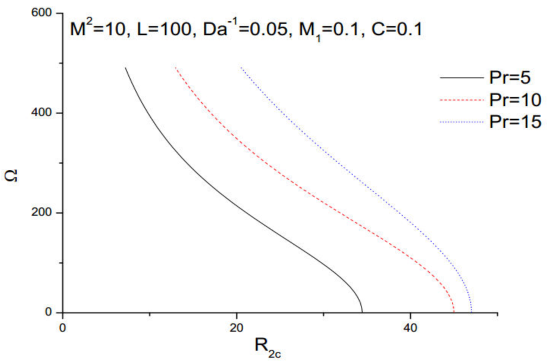

Fig 6: The graph of correction Rayleigh number R_{2c} vs Non-dimensional frequency of modulation Ω with variation in the values of Prandtl Number Pr.

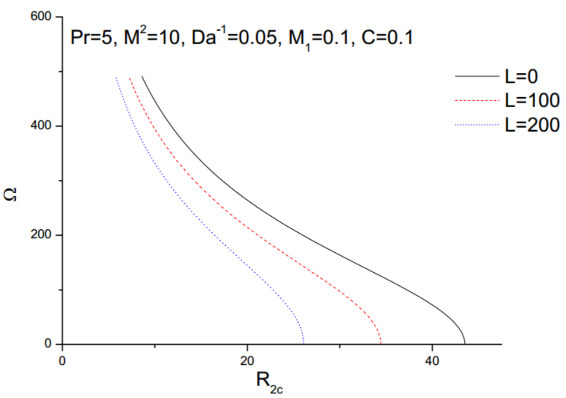

Fig 7: The graph of correction Rayleigh number R_{2c} vs Non-dimensional frequency of modulation Ω with variation in the values of Electric Rayleigh Number L.

4. CONCLUSION

Gravity modulation delays convection and weak conduction of fluid ensures stable condition for the system. But introduction of external electric field advances convection. When compared with Pranesh and Sameena Tarannum [5], the values of R_{2c} are drastically reduced in all the cases. The results of this problem are useful in conduction of experiments on couple stress fluids in space crafts.

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