Multi-criteria Generalized Assignment Problem based on Hesitant Fuzzy Weighted Geometric Operator

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Abstract

Multi-criteria Generalized Assignment Problem (MCGAP) is a very well-known topic in the real world. The purpose of the paper is to develop a method using Hesitant Fuzzy Weighted Geometric (HFWG) Operator, Score Function Matrix and Extremum Difference Method (EDM) for finding the optimal solution. An application study of the proposed method on three different criteria is conducted. The parameters of the different criteria have been considered as Hesitant Fuzzy Elements (HFEs) and have been aggregated using Hesitant Fuzzy Weighted Geometric Operator. Then the score functions of the elements have been determined. Taking them into account as the initial input data, Extremum Difference Method has been applied to get the optimal assignment. The study indicates that the proposed method does not require any complicated computation but still yields a reasonable and compromised optimal solution.

Keywords: *MCGAP; HFWG Operator; EDM.*

1. Introduction

The concept of Hesitant Fuzzy Set (HFS) has been introduced by Torra and Narukawa [5]. They applied it to describe the situation that permits the membership of an element to a given set having a few different values which arises in many real situations. HFSs are characterised by a membership function represented by a set of possible values which are able to express the hesitancy of human beings efficiently. The Hesitant Fuzzy Elements (HFEs) are the basic elements of HFS. Xu and Xia [7] gave an example to illustrate the appropriateness of HFEs and proved that the criteria of an alternative can be represented by a HFE. It is noted that the HFE can describe the situation more objectively than any crisp number or the interval valued fuzzy number or the intutionistic fuzzy number because the degree that the alternatives should satisfy the criterion are only three possible values. For the application of HFSs, S. Kar et. al [4] applied TOPSIS method to solve GAP. For the decision making problems, many useful methods [3] have been proposed to solve Hesitant Fuzzy Multi-criteria Decision Making (HFMCDM) problems. Xu and Zhang [8] used hesitant fuzzy TOPSIS approach based on maximizing deviation method to handle the HFMCDM problems with incomplete weight information. Zhang and Wei [9] applied VIKOR method to solve MCDM problems. Zhang and Xu [10] proposed an interval programming method for solving Multi-criteria Group Decision Making (MCGDM) problems. Na Chen et. al [2] solved the GDM problems applying preference relations.

Besides these, many researchers have described the application of HFS theory to Multi Attribute Decision Making (MADM) problems using weighted aggregation of the attribute values across all attributes w.r.t. each alternative to obtain an overall value. Xia and Xu [6] developed a method to deal with MADM problems with anonymity based on a series of Hesitant Fuzzy Aggregation Operators (HFAOs). Zhu et. al [11] proposed an approach to address MADM problems using the Weighted Hesitant Fuzzy Geometric Bonferroni Mean and the Weighted Hesitant Fuzzy Choquet Geometric Bonferroni Mean Operators. B. Farhadinia [1] used different aggregation operators for ranking hesitant fuzzy values to solve MADM problems.

This paper contributes a novel method for solving HFMCGAP. First, we aggregate different HFEs for different criteria of an alternative using HFWG operator. Then the score functions of the elements have been determined. Taking them into account as the initial input data, EDM has been applied to get the optimal assignment.

The structure of this paper is organized as follows- Section 2 briefly reviews some definitions, concepts and operations on HFEs, Section 3 presents the mathematical formulation of HFMCGAP and its solution methodology, Section 4 illustrates the numerical example to demonstrate the applicability and the implementation process of the proposed method. Section 5 concludes the paper.

2. Preliminaries

2.1 Hesitant Fuzzy Set (HFS)-

Definition: Let X be a reference set. A HFS A on X is defined in terms of a function $h_A(x)$ that returns a subset of [0,1] when it is applied to X i.e, $A=\{< x, h_A(x)>| x \in X\}$, where $h_A(x)$ is a set of some different values in [0,1], representing the possible membership degrees of the element $x \in X$ to A. $h_A(x)$ is called a HFE, a basic unit of HFS.

Ex:- Let $X = \{x_1, x_2\}$ be a reference set, $h_A(x_1)=\{0.3,0.4,0.5\}$, $h_A(x_2)=\{0.6,0.7\}$. Then A is a HFS s.t. $A=\{<x_1,\{0.3,0.4,0.5\}>,< x_2\{0.6,0.7\}>\}$.

2.2 Operations on HFEs-

Given three HFEs h, h_1, h_2 respectively, the basic operations of HFEs are defined as follows-

a) $h_1 \oplus h_2 = \bigcup_{\gamma_1 \in h_1, \gamma_2 \in h_2} \{\gamma_1 + \gamma_2 - \gamma_1\gamma_2\}$
b) $h_1 \otimes h_2 = \bigcup_{\gamma_1 \in h_1, \gamma_2 \in h_2} \{\gamma_1\gamma_2\}$
c) $\alpha h = \bigcup_{\gamma \in h}\{\gamma^{\alpha}\},(\alpha > 0)$
d) $h^c = \bigcup_{\gamma \in h}\{1-\gamma\}$

2.3 Some definitions

2.3.1 Score Function- Let h= $\bigcup_{\gamma \in h} \{\gamma\} = \{\gamma_j\}_{j=1}^{l(h)}$ be a HFE, where 'l(h)' returns the number of values in 'h'. A Score Function 'S' of a HFE is given by,

$$S(h) = \frac{\sum_{j=1}^{l(h)} \delta(j)\gamma_j}{\sum_{j=1}^{l(h)} \delta(j)}$$, where $\{\delta_j\}_{j=1}^{l(h)}$ is a positive value.

If we consider the sequence $\{\delta_j\}_{j=1}^{N}$, then the Score Function can be expressed as,

$$S(h) = \frac{\sum_{j=1}^{N} j\gamma_j}{\sum_{j=1}^{N} j} = \frac{2}{N(N+1)}\sum_{j=1}^{N} j\gamma_j \qquad [1]$$

Example: Let $h_1= \{0.2, 0.5, 0.8\}$, $h_2= \{0.3, 0.4, 0.8\}$ be two HFEs and it is evident that $h_1 \neq h_2$.

Now, $S(h_1) = \frac{2}{3(3+1)}(1\times 0.2 + 2\times 0.5 + 3\times 0.8)$

$= \frac{1}{6}(0.2+1.0+2.4) = \frac{3.6}{6} = 0.6$

$S(h_1) = \frac{2}{3(3+1)}(1\times 0.3 + 2\times 0.4 + 3\times 0.8)$

$= \frac{1}{6}(0.3+0.8+2.4) = \frac{3.5}{6} = 0.583$

2.3.2 Hesitant Fuzzy Weighted Geometric (HFWG) Operator

In recent years, researchers have carried out a wide study on HFE aggregation operators and their application in decision making. They developed some hesitant fuzzy operational rules on the basis of the interconnection between HFS and Intuitionistic Fuzzy Set (IFS). To aggregate HFEs the researchers proposed different operators under various situations and discussed the relationship among them.

For a collection of HFEs h_i (i= 1, 2, ………, n) and the weight vector of h_i denoted by

W= (w_1, w_2, ……..,w_n) with $w_i \in [0, 1]$ and $\sum_{i=1}^{n} w_i = 1$, $\lambda > 0$. The HFWG Operator is defined as-

$$HFWG(h_1, h_2, \ldots, h_n) = \overset{n}{\underset{i=1}{\otimes}} (h_i^{w_i}) = \Upsilon_{\gamma_1 \in h_1, \gamma_2 \in h_2, \ldots, \gamma_n \in h_n} \{ \pi_{i=1}^{n} \gamma_i^{w_i} \} \quad [2]$$

3. Hesitant Fuzzy Multi-Criteria Generalized Assignment Problem (HFMCGAP):

In this section we have formulated a real life GAP on multiple criteria under hesitant fuzzy environment through which an optimal assignment of the project to an alternative is done at minimum cost. Assume that there are four alternatives and three projects having three criteria each. Here the criteria are cost required for the project (C_1), profit expected from the project (C_2) and efficiency of an alternative (C_3) to which the project will be assigned. A decision organization including three experts (DM_1, DM_2, DM_3) is invited to assess the performance of the alternatives under each criterion and to provide the weights of the criteria. For an alternative under a criterion although all of the DMs provide their evaluation values, some of these values may be repeated. We only collect all the possible values for an alternative as a collective opinion of the DMs.

The method presented here is based on two stages. In the first stage HFWG Operator has been used to aggregate the values of different criteria of the project with respect to an alternative. In the second stage we have determined Score Function matrix. Considering this as initial input data we have solved it by EDM to get the optimal assignment. To verify the result the problem has been transformed into LPP form and solved by LINGO 9.0.

Algorithm for HFMCGAP

Step1:- Collect the values in the form of HFEs for all the criteria of the alternatives.

Step2:- Use HFWG operator to aggregate the values of different criteria for different alternatives supplied by the Decision Makers.

Step3:- Determine Score Function matrix using equation [1].

Step4:- Consider the Score Function matrix as initial input data for HFMCGAP, solve it by EDM.

Step 5:- End.

4. Numerical Example

Let us consider a Generalized Assignment Problem (GAP) consisting of three projects I, II, III and four alternatives A_1, A_2, A_3, A_4 with three criteria. The criteria are cost required for the project (C_1), profit expected from the project (C_2) and efficiency of an alternative (C_3) to which the project will be assigned. The criteria are considered as HFEs. The value of the criteria and their weight information provided by three Decision Makers (DMs) are listed in Table 1. For an alternative under a criterion, although all the DMs provide their evaluation values, some of these values may be repeated. We have collected all possible values for an alternative and listed in the input data table.

Table 1: Input data table

Project → Alternative↓	I			II			III		
	C_1	C_2	C_3	C_1	C_2	C_3	C_1	C_2	C_3
A_1	{.3,.4,.5}	{.2,.3,.5}	{.4,.4,.4}	{.1,.3,.5}	{.2,.2,.2}	{.3,.1,.2}	{.2,.3,.3}	{.1,.1,.3}	{.4,.4,.4}
A_2	{.2,.2,.2}	{.5,.1,.3}	{.4,.2,.1}	{.1,.3,.4}	{.2,.5,.1}	{.3,.4,.4}	{.1,.2,.3}	{.3,.1,.4}	{.2,.5,.1}
A_3	{.3,.3,.3}	{.4,.5,.2}	{.5,.5,.2}	{.5,.4,.4}	{.3,.2,.1}	{.4,.4,.4}	{.2,.2,.3}	{.4,.1,.2}	{.5,.4,.3}
A_4	{.2,.3,.4}	{.4,.5,.1}	{.1,.3,.5}	{.2,.2,.2}	{.3,.4,.5}	{.1,.3,.4}	{.5,.1,.1}	{.3,.3,.3}	{.1,.1,.1}
Available cost a_j→	0.60			0.80			0.70		

where for project I and alternative A_1, the data is collected as follows-

<u>Project 1</u>

Criteria ↓	Alternative →			
	DM_1	DM_2	DM_3	Collective Opinion
C_1	.3	.4	.5	{.3, .4, .5}
C_2	.2	.3	.5	{.2, .3, .5}
C_3	.4	.4	.4	{.4, .4, .4}

where a_j is the maximum available cost for the three projects.

The weights of the criteria are-

w_1 for C_1= 0.25, w_2 for C_2= 0.4, w_3 for C_3= 0.35 so that $\sum_{j=1}^{3} w_j = 1$.

Solution:

According to step 2 HFWG Operator (Equation [2]) has been used to aggregate the values of different criteria for different alternatives.

Table 2: Values obtained after applying HFWG Operator to the data of Table 1

Project → Alternative↓	I	II	III
A_1	{0.29, 0.36, 0.47}	{0.20, 0.18, 0.25}	{0.20, 0.22, 0.33}
A_2	{0.37, 0.15, 0.19}	{0.20, 0.41, 0.23}	{0.20, 0.21, 0.23}
A_3	{0.40, 0.44, 0.22}	{0.38, 0.31, 0.23}	{0.36, 0.20, 0.26}
A_4	{0.21, 0.37, 0.25}	{0.19, 0.31, 0.19}	{0.23, 0.13, 0.16}
Available cost a_j→	0.60	0.80	0.70

The aggregation value for Project 1 and alternative A_1 has been calculated by using equation [2].

HFWG (a_{11}) = $\{(0.3^{0.25}x0.2^{0.4}x0.4^{0.35}), (0.4^{0.25}x0.3^{0.4}x0.4^{0.35}), (0.5^{0.25}x0.5^{0.4}x0.4^{0.35})\}$

= {(0.74x0.53x0.73), (0.80x0.62x0.73), (0.84x0.76x0.73)}

= {0.29, 0.36, 0.47}

Now, we have determined the Score Function Matrix (according to Step 3) using Equation [1].

Table 3: Score Function Matrix

Project → Alternative↓	I	II	III
A_1	0.40	0.22	0.27
A_2	0.21	0.29	0.22
A_3	0.32	0.28	0.26
A_4	0.28	0.23	0.16
Available cost a_j→	0.60	0.80	0.70

The Score Function of a_{11} = S (a_{11}) = $\frac{2}{3(3+1)}$ (1x0.29+2x0.36+3x0.47) = $\frac{1}{6}$(2.42) = 0.40

According to Step 4, the Score Function Matrix has been considered as initial input data for HFMCGAP and has been solved by EDM-

Table 4: Solution of HFMCGAP by EDM

Project → Alternative↓	I	II	III	Row Penalties
A_1	0.40	[0.22]	0.27	0.18
A_2	[0.21]	0.29	0.22	0.08
A_3	0.32	0.28	[0.26]	0.06
A_4	0.28	0.23	[0.16]	0.12
Available cost a_j→	0.60	0.80	0.70	

Therefore the optimal assignment is-

$A_1 \to \text{II}, A_2 \to \text{I}, A_3 \to \text{III}, A_4 \to \text{III}$.

The optimal cost = 0.22+0.21+0.26+0.16 = 0.85

To verify the result the problem has been transformed into LPP form and solved by LINGO 9.0.

Min = Z;

Z= $0.40x_{11} + 0.22x_{12} + 0.27x_{13} + 0.21x_{21} + 0.29x_{22} + 0.22x_{23} + 0.32x_{31} + 0.28x_{32} + 0.26x_{33} + 0.28x_{41} + 0.26x_{42} + 0.16x_{43}$;

$C_1 = x_{11} + x_{12} + x_{13}$; $C_1 = 1$;

$C_2 = x_{21} + x_{22} + x_{23}$; $C_2 = 1$;

$C_3 = x_{31} + x_{32} + x_{33}$; $C_3 = 1$;

$C_4 = x_{41} + x_{42} + x_{43}$; $C_4 = 1$;

$C_5 = 0.40x_{11} + 0.21x_{21} + 0.32x_{31} + 0.28x_{41}$; $C_5 \le 0.60$;

$C_6 = 0.22x_{12} + 0.29x_{22} + 0.28x_{32} + 0.23x_{42}$; $C_6 \leq 0.80$;

$C_7 = 0.27x_{13} + 0.22x_{23} + 0.26x_{33} + 0.16x_{43}$; $C_7 \leq 0.70$;

Solving by LINGO 9.0 assignment is $A_1 \rightarrow \text{II}, A_2 \rightarrow \text{I}, A_3 \rightarrow \text{III}, A_4 \rightarrow \text{III}$.

Optimal value cost = 0.22+0.21+0.26+0.16 = 0.85

5. Results and Discussions

This paper explains how to solve MCGAP under hesitant fuzzy environment. Here we have considered the criteria values and weights of different criteria as HFEs supplied by three DMs. Considering their collective opinion, we have aggregated the values using HFWG operator. On the basis of these aggregation values, Score function matrix has been determined which is considered as initial input data.The problem is then solved by EDM. To verify the results the problem has been transformed into LPP form and solved by LINGO 9.0.

The proposed method solves the Multi-criteria GAP in the easiest way and very efficiently. It can solve the problem if number of criteria increases. The problem can also be solved under fuzzy, intuitionistic fuzzy, interval valued fuzzy set etc.

6. Conclusions

HFS is characterised by a membership function including a set of possible values which is a new effective tool to express people's hesitancy in daily life. In this paper, we have presented a novel approach for solving MCGAP by applying HFWG Operator. For this, all the criteria values have been aggregated followed by determining the Score function matrix. Finally, EDM has been used to get the optimal assignment. The proposed method for solving MCGAP under hesitant fuzzy environment where number of criteria markedly exceeds the number of alternatives does not require complicated computation procedures but still yields a reasonable and credible solution. Hence, it is a very flexible, robust and effective method to solve MCGAP.

References

[1] B. Farhadinia, A Novel Method of Ranking Hesitant Fuzzy Values for Multiple Attribute Decision-Making Problems, International of Intelligent Systems, Vol. 00, pp. 1-16 (2013).

[2] Na Chen, ZeshuuiXu, M. Xia, Interval-valued Hesitant Preference Relations and their Applications to Group Decision-Making, Knowledge Based Systems, Vol. 37, pp. 528-540 (2013).

[3] R. M. Rodriguez, L. Martinez, V. Torra, Z.S. Xu, F. Herrera, Hesitant Fuzzy Sets: State of the Art and Future Directions, International Journal of Intelligent Systems, Vol. 29, Issue 6, pp. 495-524 (2014).

[4] S. Kar, K. Basu, S. Mukherjee, Generalized Fuzzy Assignment Problem with restriction on the cost of job under hesitant fuzzy environment,OPSEARCH, Springer, Volume 52, Issue 3, September, 2015, pp. 401-411.

[5] V. Torra, Y. Narukawa, On hesitant fuzzy sets and decision, The 18th IEEE International Conference on Fuzzy systems, Jeju Island Korea, 2009, pp.1378-1382.

[6] Xia M. M., Xu Z. S., Hesitant Fuzzy Information Aggregation in Decision-Making, International Journal of Approximate Reasoning, Vol. 52, pp. 395-407 (2011).

[7] Xu Z. S., Xia M. M., Distance and Similarity Measures for Hesitant Fuzzy Sets, Information Sciences, Vol. 181, pp. 2128-2138 (2011).

[8] Xu Z. S., Zhang X. L., Hesitant Fuzzy Multiple Attribute Decision-Making based on TOPSIS with Incomplete Weight Information, Knowledge Based Systems, Vol. 52, pp. 53-64 (2013).

[9] Zhang N., Wei G. W., Extension of VIKOR method for Decision-Making Problem based on Hesitant Fuzzy Set, Applied Mathematical Modeling, Vol. 37, pp. 4938-4947 (2013).

[10] Zhang X. L. and Xu Z.S., Interval Programming Method for Hesitant Fuzzy Multiple Attribute Group Decision-Making with incomplete preference over alternatives, Computers and Industrial Engineering, Vol. 75, pp. 217-229 (2014).

[11] Zhu B., Xu Z., Xia M. M., Hesitant Fuzzy Geometric Bonferroni means, Information Science, Vol. 205, pp. 72-85 (2012).