Neutron Leakage Rate in Spheroids of Varying Axis Ratios

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Abstract

Neutrons leakage rates in spheroids are investigatedas a function of their axis ratios. First, the Neutron diffusion equation is derived from Fick's law of diffusion, thereafter; the geometric buckling for a spherical nuclear reactor is obtained. The geometric buckling for Prolate and Oblate spheroids are obtained as a function of axis ratio of the spheroid. Secondly, the neutron thermal lifetime is expressed in terms of geometric buckling term resulting in the establishment of a relationship between the geometric buckling and axis ratio of the spheroid. The relation is then used to derive the desired equation of neutron leakage rate versus the axis ratio for the spheroid. Finally, popular software is used to obtain the results. The relationship shows that for the same value of axis ratio(c) and the neutron lifetime constant (k), the neutron leakage rate for an Oblate spheroid is slightly higher than that of Prolate spheroids. This is due to the fact that the surface area of an Oblate spheroid is much larger than the surface area of a Prolate spheroid for the same volume. Furthermore; the relationship is seen to be proportional and largely dependent on k . It was also found that at large axis ratio; there is little dependence by the neutron leakage rate on the axis ratio.

Key Words:*Neutron Leakage; Prolate; Oblate, axis Ratio; Geometric Buckling.*

1. Introduction

The neutron flux inside and outside a reactor core plays a significant role in nuclear theory. As a result, the concept of neutron diffusion and moderation has been critically examined in order to understand the behavior of neutron in a nuclear reactor core[1,2,3]. The first concept that was made in this study is the derivation of geometric and materials buckling of nuclear reactors. The geometric buckling for various nuclear reactor geometries has been greatly considered in[1,2,3,4,5]. The geometric buckling for nuclear reactor core for spheroids was discussed in detail[4], spherical geometry[1,2,4,9], cylindrical geometry[3,4]. The criticality of a reactor is also a major component in reactor concept. The materials buckling of a reactor together with its geometric buckling determines the criticality of a reactor and are discussed[6,7]. For spheroids, the main determinant of its geometric buckling is the axis ratio of that spheroid. Consequently, this affects the behavior of neutron diffusion in both Prolate and Oblate spheroids. The determination of axis ratios of spheroids in relation to their geometric buckling is considered[4,8,9]. Furthermore, as neutrons diffuse from the core of the reactor towards the surface of a spheroid, the neutrons are bound to leak as determined in neutrons diffusion equation[4,7,8]. The thermal neutron lifetime determines how neutrons leak from anuclearreactor of various geometric buckling factors. The thermal neutron lifetime is discussed[6].

2. Background to the Neutron Diffusion Problem

Since neutrons do not disappear (B-decay is), the following must be true for an arbitrary volume (V):

[Rate of change in number of neutrons in V]= [Rate of production of neutrons in V] - [Rate of absorption ofNeutrons in V] - [Rate of leakage of neutrons in V].

In mathematical terms, at steady state condition, the equation of continuity or the equation of neutron balance can be expressed as;

Neutrons change rate, $N_c = \int_v \frac{\partial j}{\partial t} dV$, (j is the neutrons density),

Neutrons supply rate, $X = \int_v S dV$, (S is the rate at which neutrons are supplied to the reactor core),

Neutrons absorption rate, $A_R = \int_v \Sigma_a \phi \, dV$, ($\sum_a$ macroscopic neutron absorption cross section),

Leakage rate, $L_R = \int_v \nabla j dV$.

We now consider the **fig 1** and proceed to obtain the geometric buckling of the reactor,

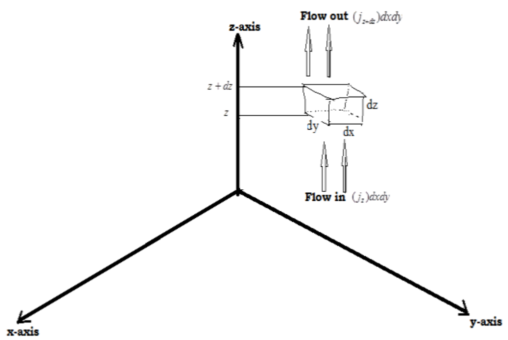

Fig 1: Neutron diffusion flux inside a nuclear reactor core.

We start by isolating an infinitesimal small cube of volume, dv, as shown in **fig 1**. The volume element, dv, is given by:

$$dv = dxdydz \tag{1}$$

In each of the surfaces of the cube, a net neutron current, j exists. Assuming that we consider only one surface perpendicular to the z-axis. The difference between the neutrons current densities[4,6] is given by

$$j_{z+dz} - j_z = (\frac{dj}{dz})_z dz \tag{2}$$

Based on Fick's' law of diffusion according to which the solute will diffuse from high concentration to low concentration[1,4] the neutron current density is given by;

$$j = -D\nabla\phi \tag{3}$$

Where $D_x = D_y = D_z = D$ is the neutron diffusion coefficient for an isotropic system, and ϕ is the neutron flux

Combining the eqs(1), (2) and (3) we obtain,

$$j = -\frac{d}{dz}\left(D\frac{d\phi}{dz}\right)dxdydz \tag{4}$$

The total leakage rate per unit volume, L_v, is given by;

$$L_v = -D\left[\left(\frac{d^2\phi}{dx^2}\right) + \left(\frac{d^2\phi}{dy^2}\right) + \left(\frac{d^2\phi}{dz^2}\right)\right] \tag{5}$$

Therefore,

$$-D\left[\left(\frac{d^2\phi}{dx^2}\right) + \left(\frac{d^2\phi}{dy^2}\right) + \left(\frac{d^2\phi}{dz^2}\right)\right] = -D\nabla^2\phi \tag{6}$$

We now move to consider a nuclear reactor core. From the previous discussions[1,4,6], it was noted that neutrons supply to the reactor, (X **)**, is at a rate given by the sum of Neutrons leakage rate (L_R) and neutron absorption rate (A_R).

At a steady state condition, the neutrons change rate, N_c, the equation of neutron balance will be given by

$$X = L_R + A_R \tag{7}$$

The equation for neutron absorption rate is given by:

$$A_R = \Sigma_a \phi \tag{8}$$

Therefore, neutron diffusion equation will be given by:

$$D\nabla^2\phi - \Sigma_a \phi + X = 0 \tag{9}$$

We can write the neutron supply rate to the reactor as,

$$X = v\Sigma_f \phi \tag{10}$$

Σ_f is the fission cross section while v is the neutron fission fraction. Eq(9) becomes;

$$D\nabla^2\phi - \Sigma_a \phi + v\Sigma_f \phi = 0 \tag{11}$$

Therefore, eq (2.11) finally reduces to;

$$\nabla^2\phi + \frac{(v\Sigma_f - \Sigma_a)\phi}{D} = 0 \tag{12}$$

The geometric buckling factor term, is given by,

$$B_g^{\,2} = \frac{(v\Sigma_f - \Sigma_a)}{D} \tag{13}$$

Finally,eq (12) can be written as,

$$\nabla^2\phi + B_g^{\,2}\phi = 0 \tag{14}$$

Equation (14) is the desired neutron diffusion equation at steady state conditions.

2.1 Geometric Buckling for Spherical and Spheroids

It should be noted that for a critical reactor, the materials buckling factor (B_m) which exclusively depends on thematerials composition of the core is related to the geometric buckling, (B_g).

$$B_m^2 = B_g^2 \quad (15a)$$

The geometric buckling of a nuclear reactor depends on the geometry of the reactor core[2,3,5,9]. The geometric buckling of a spherical reactor is obtained by solving theeq (2.14) in spherical coordinates to give,

$$B_g{}^2 = \left[\frac{\pi}{R}\right]^2 \quad (15b)$$

Where R is the radius of the sphere,

For spheroids, the geometric buckling is obtained by solving the eq(14) in spheroid geometry by using the transformations in eqns 16(a)-16(d);

$$x = \frac{d}{2}\sqrt{(1-\eta^2)(1+\xi^2)}\cos\phi \quad (16a)$$

$$y = \frac{d}{2}\sqrt{(1-\eta^2)(1+\xi^2)}\sin\phi \quad 2.16(b)$$

$$z = \frac{d}{2}\eta\xi \quad 2.16(c)$$

Where, $\eta \in [0,1], \xi \in (-\infty,\infty)$, $\phi \in (0,2\pi)$ (16d)

The reactor equation forProlate spheroid[4]is stated as;

$$\frac{\partial}{\partial\xi}\left[(\xi^2-1)\frac{\partial\psi}{\partial\xi}\right] + \frac{\partial}{\partial\eta}\left[(1-\eta^2)\frac{\partial\psi}{\partial\eta}\right] + \frac{\xi^2-\eta^2}{(\xi^2-1)(1-\eta^2)}\frac{\partial\psi}{\partial\phi} + B_g{}^2 a^2(\xi^2-\eta^2)\psi = 0 \quad (17)$$

While for Oblate spheroid, the equation is;

$$\frac{\partial}{\partial\xi}\left[(\xi^2+1)\frac{\partial\psi}{\partial\xi}\right] + \frac{\partial}{\partial\eta}\left[(1-\eta^2)\frac{\partial\psi}{\partial\eta}\right] + \frac{\xi^2+\eta^2}{(\xi^2+1)(1-\eta^2)}\frac{\partial\psi}{\partial\phi} + B_g{}^2 a^2(\xi^2+\eta^2)\psi = 0 \quad (18)$$

The eqns (17) and (18) are solved through separation of variables[4]to yield a relation for geometric buckling (B_g) for Prolate core, the geometric buckling is given by,

$$B_g^2 M_2^2 = \frac{\pi^2}{3}\frac{2c^2+1}{c^2} \quad (19)$$

Secondly, for Oblate core, the geometric buckling is given by,

$$B_g^2 M_2^2 = \frac{\pi^2}{3}\left\{\frac{c^2+2}{c^2}\right\} \quad (20)$$

The values of geometric buckling for other geometries is summarized in the Table 1

Table1: showing Geometric Buckling values for various Geometries

	Geometry	Geometric Buckling
1	Cylindrical	$\left[\frac{\pi}{R}\right]^2$ R is the radius of the Core
2	Spherical	$\left[\frac{\pi}{R}\right]^2 + \left[\frac{2.405}{R}\right]^2$ R is the radius of the cylinder
3	Infinite plane	$\left[\frac{\pi}{a}\right]^2$ a is the length of the plane
4	Cartesian	$\pi^2\left\{\left[\frac{1}{a}\right]^2 + \left[\frac{1}{b}\right]^2 + \left[\frac{1}{c}\right]^2\right\}$ a,b,c are the dimensions of the rectangular core

3. Methods of Solutions

3.1 Axis Ratio-Neutron leakage ratesfor Prolate spheroid

For a Prolate spheroid, the neutron thermal lifetime is given as

$$l_t = \frac{l_0}{(1+B_g^2 L^2)} \quad (21)$$

Or

$$l_t(1+B_g^2 L^2) = l_0 \quad (22)$$

Making B_g the subject of the formula, we obtain,

$$B_g^2 = \frac{l_0 - l_t}{L^2 l_t} \quad (23)$$

Further, we let

$$T = \frac{l_0 - l_t}{l_t} \quad (24)$$

Substituting eq (23) in eq (22) we obtain,

$$B_g^2 = \frac{T}{L^2} \quad (25)$$

The derivation of geometrical buckling for a Prolate spheroid[4] is defined according to equation;

$$c = \left[\frac{\pi^2}{3B_g^2 M_2^2 - 2\pi^2}\right]^{1/2} \quad (26)$$

From equation eq (25)we can rewrite equation eq (26) as

$$c = \left[\frac{\pi^2}{3\frac{T}{L^2} M_2^2 - 2\pi^2}\right]^{1/2} \quad (27)$$

We also introduce the neutron thermal lifetime constant,k,given by,

$$k = 3M_2^2 T$$

Therefore, eq (27) becomes,

$$c = \left[\frac{9.86L^2}{k - 19.72L^2}\right]^{1/2} \quad (28)$$

The Neutron leakage rate versus axis ratio of the Prolatespheroid is given as,

$$L = \left[\frac{kc^2}{9.86 + 19.72c^2}\right]^{1/2} \quad (29)$$

3.2 Axis Ratio-Neutron leakage rates for Oblate spheroid

The geometric buckling for Oblate spheroid is defined according to (20);

Recalling equation (3.04), for geometric buckling

$$B_g^2 = \frac{T}{L^2}$$

The eq (20) can be rearranged as;

$$B_g^2 M_2^2 c^2 = \frac{\pi^2}{3}\{c^2 + 2\} \quad (30)$$

Finally, we obtain the relation

$$c = \left[\frac{19.72L^2}{k - 9.86L^2}\right]^{1/2} \quad (31)$$

The Neutron Leakage Rate versus Axis ratio of the Oblatespheroid is given as;

$$L = \left[\frac{kc^2}{19.72 + 9.86c^2}\right]^{1/2} \quad (32)$$

Where; Neutron thermal lifetime= l_t, Neutron thermal lifetimewhen there is no leakage= l_0, Neutron leakage rate= L, Semi-minor axis= M_2, Semi-major axis= M_1 and Neutron and neutron thermal life time constant = k

4. Results and Discussions

Fig2 Shows the Neutron leakage rate versus axis ratio of anOblatespheroid at various values of k.

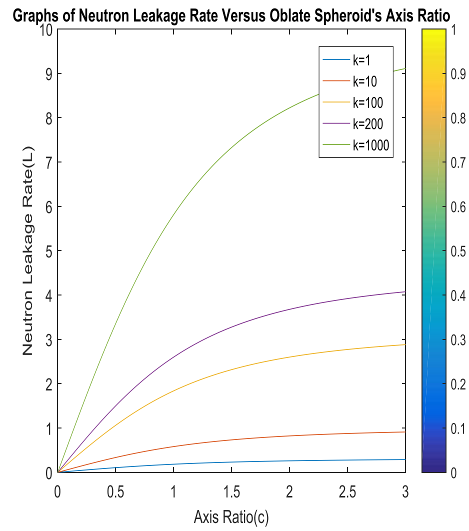

Fig2:Neutron leakage rate versus axis ratio of aOblatespheroid at various values of k

Figure 3 Shows Neutron leakage rate versus axis ratio of anProlatespheroid at various values of k.

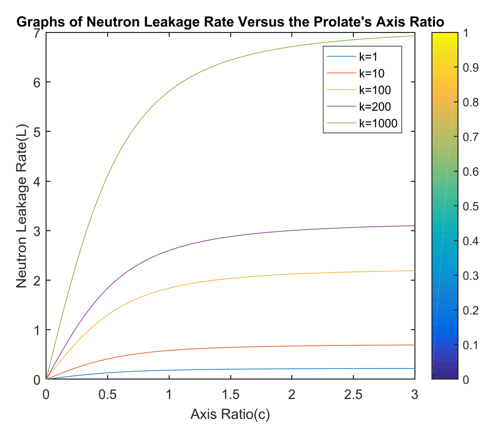

Fig 3: Neutron leakage rate versus axis ratio of aProlatespheroid at various values of k

Figure 4 shows Neutron Leakage rate versus the Axis Ratio of the Spheroid at various values of k.

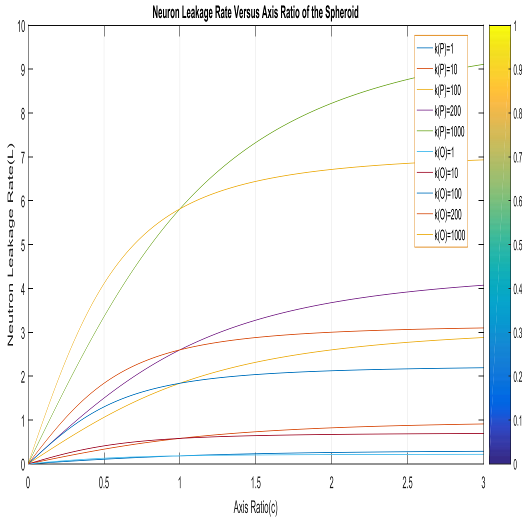

Fig 4: Neutron Leakage rate versus the Axis Ratio of the Spheroid at various values of k

NB:(k (O) is the k value for Oblate Spheroid while k(P) is the k value for Prolate Spheroid)

The results show that the behavior for the leakage rate of neutrons for both Prolate and Oblate spheroids possess great similarity as depicted in **fig2**, **fig 3** and **fig4**. The figures show two sections of neutrons leakage rate as the axis ratio of the spheroid increases; the first section is when the axis ratio is between 0 and 1. In this section, the neutron leakage rate for the Oblate Spheroid is always higher than that of Prolate spheroid for a given value of k .Thisisbecauseat small axis ratios, the surface area of an Oblate spheroid is much larger than the surface area of a Prolate spheroid with the same volume.Subsequently, as the axis ratio becomes greater than 1, the neutron leakage rate for Prolate spheroid overtakes that of Oblate spheroid. The probable explanation for this is that at such large axis ratios, the surface area of the Prolate spheroid becomes larger than that of Oblate spheroids of the same volume. It was also observed that when the axis ratio is 1, the neutron leakage rate for the Oblatespheroidand Prolate spheroid converges since all values of this point represent the special case of sphere. Thirdly, it was found out that at large axis ratio, there is little dependence of the neutron leakage rate on the axis ratio of the spheroid. The explanation given is that at such axis ratios, the probability of neutrons with large energy values reaching the surface of the reactor core has significantly reduced, consequently, the probability of neutron leakage also reduces. Lastly, we found out that at large value of (k) e.g. **k** >1000, the neutron leakage rate for both Prolate and Oblate spheroids almost superimpose. This is because for large neutron leakage constant, k, the other terms in equations equations(3.10) and (3.13) tend to converge for both Oblate and Prolate spheroids respectively.

5. Conclusions

The relationship between neutron leakage rate and spheroid axis ratio has been developed. The resulting relationship is used to explain the behavior of neutron diffusion in both Prolate and Oblatespheroids. For both Prolate and OblateSpheroids of the same volumes, the neutron leakage rate versus the axis ratio exhibit the same behavior though the neutron leakage rate for Prolate spheroids is lower compared to that of Oblate spheroids of the same value of axis ratio for small axis ratios. However, for large axis ratios, it was found that the neutrons leakage rate for Prolate spheroids becomes greater than that of an oblate spheroid of the same volume. This is because at small axis ratios, the surface area of an Oblate spheroid is much larger than the surface area of a Prolate spheroid with the same volume. Atlarge axis ratio, there is little dependence by the neutron leakage rate on the axis ratio of the spheroid. Furthermore, the values for neutron leakage rate for both Prolate and Oblate spheroids are almost of the same value.

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