Effect of Magnetic Field on the Properties of High Temperature Superconductors

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Abstract

The effect of the magnetic field (H) on a 2-D square lattice and effective magnetic flux (Φ) per plaquette has been studied. Magnetic flux quantization provides evidence for existence of Cooper pairs, thus there exists magnetic flux in a superconducting material, but of strength that will not destroy superconductivity. Consequently magnetic flux mediated interactions act as a pairing glue. For an external applied magnetic field on a 2-D square lattice calculations show that forH_{c_2} = 3000000G and a = 2.3Å, the value ofα = 2.25 x 10^{-7} and T_c = 300K (room temperature). The effective magnetic flux per plaquette was found to increase with increase in T_c and its value was 5.84G/m^2 when$T_c = 300K$.

Key words: *Applied Magnetic Field, Effective Magnetic Flux per plaquette, Transition Temperature.*

1. Introduction

The effect of magnetic field on the superconducting state has been a subject of intense study since the discovery of the so-called Meissner effect by Meissner and Ochsenfeld in 1933[1]. They found that the magnetic field in a superconducting tin and lead does not penetrate into the bulk of a superconductor, but is rather confined to a surface layer of thickness λ, called the London penetration depth[2] the size of λ is roughly of the order of tens to hundreds of manometers[3]. But they found that below some critical temperature T_c, the flux was expelled from the bulk of the superconductor. However, if the superconducting sample is not simply connected, i.e., if it has holes, such as in the case of a superconducting ring, then in the Meissner phase, the flux maybe trapped in the holes. Such trapped flux is quantized in integer units of superconducting fluxoid Φ_L= 2.07 x $10^{-7} Gcm^2$. The Meissner state exists for some critical temperature T_c when the applied magnetic field H is smaller than some critical magnetic field H_c(T). Such materials are called type-I superconductors. For magnetic field more than H_{c_1}(T), there is uniform flux penetration into the system or sample the system changes from the superconducting to the normal state. Then there are type-II superconducting materials for which the critical magnetic field that reverts them from the superconducting to the normal state is higher, and this is denoted by $H_{c_2} > H_{c_1}$. In 1935, Fritz and Heinz London proposed a theory for the macroscopic behaviour of superconductors and studied the effect of an applied external magnetic field to describe the super flow. The flux quantum was given by,Φ_L=$\frac{hc}{2e}$, and this was half the size of the Dirac flux quantumΦ_D=$\frac{hc}{e}$. This was confirmed in the experiments[4-6].

Disappearance of superconductivity under the action of large magnetic field lead to the concept that superconductivity and ferromagnetism cannot co-exist[7]. But later some superconductors were discovered in which the superconducting state and ferromagnetism was found to co-exist[8]. The superconducting materials that showed co-existence of superconductivity and ferromagnetism are $LaAlO_3$ and $SrTiO_3$[9] and iron based pnictides[10]. Recently [11] experiments were done to study the effects of pressure and magnetic field on the superconductivity in $ZrTe_3$. It was found that the upper critical magnetic field can be associated with two different types of suppression mechanism of superconductivity(SC), and these mechanisms are Pauli paramagnetism orbital effects. The superconductivity is induced by local and Cooper pairs. Several theoretical models have predicted that in local pair-induced superconductivity, the H_{c_2} is determined by the Pauli paramagnetic effect. Thus the spin of the electron, the orbital motion of the electron, and the associated magnetic moments, including the magnetic moment of the Cooper pair, have a definite role to play in determining superconducting state. Hence, under the action of external magnetic field, the magnetic moments and flux trapping will play an important role in determining the transition temperature T_c to the superconducting state. In our study we have studied the effect of the applied magnetic field, the flux trapping on the superconducting transition temperature T_c.

2. Magnetic Field Effects on Superconductors

Superconductivity in metals is a consequence of formation of Cooper pairs from electrons. Even at a temperature T less than the transition temperature T_c of the given superconductor, Cooper pairs can be destroyed by the application of external magnetic fields through two effects.

The first is called the diamagnetic effect, in which increasing the magnetic field, H, will change the orbital motion of Cooper pairs and increase their energy. When the applied magnetic field is stronger than the critical field B_c, this increase in energy becomes higher than 2Δ (2Δ is the energy required to form the Cooper pair). As a result, the electrons do not prefer forming Cooper pairs.

The second is called the paramagnetic effect, in which all the electrons can lower their energy by aligning the electron magnetic moments (electron magnetic moments are due to the spin of the electron) parallel to the magnetic field instead of forming Cooper pairs with opposite spins.

3. Ising Superconductors

Recently, a type of superconductor called Ising superconductors was discovered. These superconductors can survive even when the applied magnetic field is as strong as 60 Tesla, compared to the largest magnetic fields which can be created in the laboratory. Hence Ising superconductors can overcome both the paramagnetic and diamagnetic effects of the magnetic fields, and superconductivity survives even when the applied magnetic field is very large.

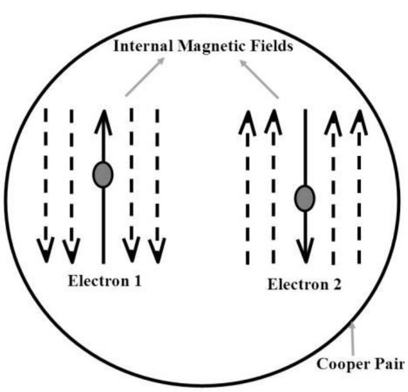

Fig 1. Cooper pair with internally generated magnetic fields.

Two electrons form a Cooper pair with spin and momentum in the opposite directions. Electron 1 with momentum **p** experiences internal magnetic field $\boldsymbol{B}_{1\perp}$= (0,0, -B_z) but electron 2 with momentum -**p** experiences an opposite magnetic field $\boldsymbol{B}_{2\perp}$= (0,0, B_z) the magnetic fields are denoted in dashed arrows.

In Ising superconductors, internal magnetic fields of the order of 100 Tesla are created and thus applied magnetic fields of order of 70 Tesla cannot destroy the superconducting state. It's something like saying that the superconductivity is switched on by the internal magnetic field in an Ising superconductor.

It has been recently discovered experimentally and proposed theoretically[12] that the lattice structure of MOS_2 thin films allows the moving electrons in the material to experience strong internal magnetic fields of about 100 Tesla. This special type of internal magnetic fields, instead of damaging superconductivity, protects the superconducting electron pairs from being destroyed by external magnetic fields. They called this type of superconductors, 'Ising superconductors'. They also predict that many other superconductors, which have similar lattice structure as MOS_2, would fall into the same family of 'Ising superconductors' as well [13, 14]. This means there could be superconductors with large internal magnetic moments, dipole moments and magnetic fields.

4. Theoretical Calculations

A common feature of all superconductors, both the low and high temperature superconductors, is that the electrons somehow form Cooper pairs. Evidence for Cooper pairs can be obtained from the observation of the magnetic flux quantization being $h/_{2e}$[15] and not $h/_e$ since a Cooper pair contains two electrons. It is also known that the main stage of pairing is the copper oxide plane. The phonon-based pairing mechanism that worked for the BCS type superconductors does not seem to be very successful for HTSC. This leads to proposal of other models with different kind of "glue" that binds the electron pairs. Because of the proximity to anti-ferromagnetic regime it has been proposed that instead of phonons, magnetically mediated interactions could act as a pair glue[16]. It was proposed[17] that superconductivity could arise from doping of spin – $\frac{1}{2}$particle singlets, resulting into quantum fluctuations that could destroy the anti-ferromagnetics (AF) phase, resulting into a liquid of singlet pairs. This provides a close link between the pairing for superconductivity and the magnetism in the sample. It is, therefore, assumed that there exists dipole-dipole interactions, dipole-electron interactions and thus there exist inherent magnetic fields that affect the properties of a superconductor. Without destroying superconductivity as in the 'Ising superconductors'.

Suppose now we consider a two-dimensional square-lattice of spacing a, immersed in a uniform magnetic field, H, perpendicular to it as shown in Fig 2.

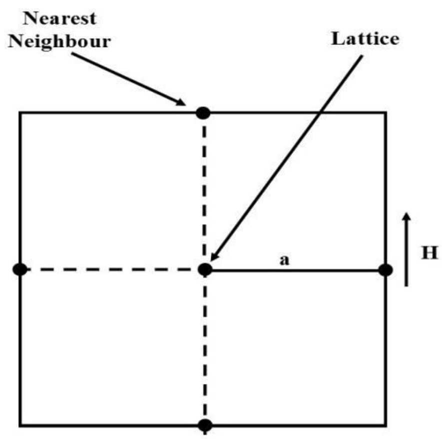

Fig 2. Application of magnetic field H to a 2D square lattice.

Consider an electron with its four immediate neighbours. The electron will undergo two periodic motions. If t is the period of motion of an electron in a state with a crystal momentum p.
Then

$p = \frac{2\pi\hbar}{a}$ since $p.a = h$ and $h = 2\pi\hbar$(1)

If t is the time taken for the electron to travel through a distance a (lattice spacing) say with velocity v, then $t = \frac{a}{v}$. The momentum p of electron is given by $p = 2\pi\hbar = mv$ *or* $v= \frac{2\pi\hbar}{ma}$.
Then

$$t = \frac{a}{v} = \frac{ma^2}{2\pi\hbar} \qquad (2)$$

If τ is the characteristic period of an electron under uniform applied magnetic field H, then the time required by an electron to move once on a circular path is τ and if the cyclotronfrequency is$f$ then

$$f = \frac{eH}{mc} \qquad (3)$$

And

$$\tau = \frac{1}{f} \qquad (4)$$

Then

$$\tau = \frac{mc}{eH} \qquad (5)$$

The ratio of t to τ is written as using equations (2) and (5) α such that

$$\alpha = \frac{t}{\tau} = \frac{a^2H}{\left(2\pi\hbar c/_e\right)} = \frac{a^2H}{\Phi_0} = \frac{\text{Flux through the lattice cell}}{\text{one Flux quantum}}$$

(6)

Where$\Phi_0 = \frac{2\pi\hbar c}{e}$ (one flux quantum).

A subatomic particle such as an electron has wave-like and particle-like properties. Its wavelength λ being related to the momentum p of the particle by the deBroglie equation $\lambda= \frac{h}{p}$. From this wave-particle duality, following uncertainty principle, with position displacement Δx and time displacementΔt are related as, Δ x $\Delta p \simeq h$, where $\Delta E \Delta t \simeq h$.the energy gap$\Delta$ is written in terms of transition temperatureT_c as,$\Delta \simeq K_B T_c$ because Cooper pairs break up at T_c. If formation of Cooper pairs is considered, bearing in mind the uncertainty principle, and assuming that the scale of time to establish Cooper pair is given by t, then

$$t \simeq \frac{\hbar}{\Delta} = \frac{\hbar}{K_B T_c} \qquad (7)$$

Superconductivity cannot be observed if the time for formation of a Cooper pair is shorter than t. Distance $\xi$by which the electron moves to form the is Pippard coherence length[18] and is given by $\xi = tv_F$. The time taken by an electron to move through the lattice of size a with velocity v is given by $t_1 = \frac{a}{v}$. For the case of applied uniform magnetic field H on $a$two-dimensional square lattice using Equations (7) and (6), we get$t = a^2H$.

$$a^2H = \frac{h}{K_B T_c} \qquad (8)$$

And

$$T_c = \frac{h}{a^2 H K_B} \text{ and } a = \sqrt{\frac{h}{k_B T_c H}} \qquad (9)$$

Now consider Harper's equation[19]

$$\frac{\epsilon+4}{\gamma}F(x)=x^2F(x) - F''(x) - \frac{\gamma}{12}[F'''(x) + x^4F(x)]$$

(10)

where$x = n\gamma^{\frac{1}{2}}$, $\gamma = 2\pi\Phi$, Φ = effective magnetic flux per plaquette, x = electronic density, ϵ = energy corresponding to one particle spectrum.

Then

$$\epsilon F(x) = -4F(x) + \gamma x^2 F(x) - \gamma F''(x) - \frac{\gamma^2}{12}[F'''(x) + x^4F(x)] \qquad (11)$$

$$\epsilon = -4F(x) + \gamma\left(2a^\dagger a + 1\right) F(x) - \frac{\gamma^2}{12}\left[\frac{d^4}{dx^4}F(x) + x^4F(x)\right] \qquad (12)$$

On calculating the expectation value of Eq. (12), we write,

$$\langle F(x)|\epsilon|F(x)\rangle = 4\langle F(x)|F(x)\rangle + \gamma(2 \quad a^\dagger a + 1$$
$$\langle F(x)|F(x)\rangle - \frac{\gamma^2}{12}\left\langle F(x)\left|\frac{d^4}{dx^4} + x^4\right|F(x)\right\rangle \quad (13)$$

or

$$E_n = -4 + \gamma(2n+1) - \frac{\gamma^2}{12}\langle F_n\left|\frac{d^4}{dx^4} + x^4\right|F_n\rangle \quad (14)$$

where $\langle F(x)|F(x)\rangle = 1$ due to normalization and $a^\dagger a = n$.

From Eq. (14) $\left(\frac{d^4}{dx^4} + x^4\right)$ is the perturbation term. To calculate the perturbation term, we substitute for x and $\frac{d}{dx}$ which are $x = \frac{1}{\sqrt{2}}(a + a^\dagger)$ and $\frac{d}{dx} = \frac{1}{\sqrt{2}}(a - a^\dagger)$ in Eq. (14) to get,

$$\frac{d^4}{dx^4} + x^4$$
$$= \left[\frac{1}{\sqrt{2}}(a - a^\dagger)\right]^4 + \left[\frac{1}{\sqrt{2}}(a + a^\dagger)\right]^4$$
$$= \frac{1}{2}\left[(a^\dagger)^4 + aaa^\dagger a^\dagger + aa^\dagger aa^\dagger + a^\dagger aaa^\dagger + aa^\dagger a^\dagger a + a^\dagger aa^\dagger a + a^\dagger a^\dagger aa\right] \quad (15)$$

Combining Eqs. (14) and (15) and using quantities in (15) we get,

$$a^\dagger\left|F_n\right\rangle = (n+1)^{\frac{1}{2}}\left|F_{n+1}\right\rangle \quad \text{And}$$
$$a\left|F_n\right\rangle = n^{\frac{1}{2}}\left|F_{n-1}\right\rangle \quad (16)$$

Using equations (15) and (16), the expectation value of Eq. (14) becomes,

$$\langle F_n\left|\frac{d^4}{dx^4} + x^4\right|F_n\rangle = \frac{1}{2}\left[(n+1)(n+2)(n+1)^2 + \left(n(n+1)\right)^2 + n^2 + n(n-1)\right]$$

$$= \frac{3}{4}\left[(2n+1)^2 + 1\right] \quad (17)$$

Using Eqs. (17) in (14) gives,

$$E_n = -4 + \gamma(2n+1) - \frac{\gamma^2}{16}\left[(2n+1)^2 + 1\right] \quad (18)$$

At the temperatures of interest, it is necessary to consider the energy difference between states in which hopping electron is on one site and then when it is on another site of similar symmetry. The difference in energy levels of the two sites gives the probability amplitude Green's function, which, according to Quantum treatment of lattice vibrations is equivalent to the thermal activation factor $e^{-\frac{\Delta E}{kT}}$. We now multiply Eq. (18) with the thermal activation factor and this gives, say E, such that,

$$E = \left[-4 + \gamma(2n+1) - \frac{\gamma^2}{16}\{(2n+1)^2 + 1\}\right]e^{-\frac{\Delta\epsilon}{kT}} \quad (19)$$

The specific heat $C_v = \left(\frac{\partial E}{\partial T}\right)$

$$= \frac{\partial\left(-4 + \gamma(2n+1) - \frac{\gamma^2}{16}[(2n+1)^2+1]e^{-\frac{\Delta\epsilon}{kT}}\right)}{\partial T} = -\frac{\gamma^2}{16}\left[(2n+1)^2 + 1\right]\left(\frac{\Delta\epsilon}{k}\right)\left(-\frac{1}{T^2}\right)e^{-\frac{\Delta\epsilon}{kT}} \quad (20)$$

The transition temperature T_c is given by,

$$\left(\frac{\partial C_v}{\partial T}\right)_{T=T_c} = 0$$

To calculate T_c, we write,

$$\frac{\partial C_v}{\partial T} = -\frac{\gamma^2}{16}\left[(2n+1)^2 + 1\right]\left(\frac{\Delta\epsilon}{k}\right)\left(\frac{-2}{T^3}\right)e^{-\frac{\Delta\epsilon}{kT}} + \frac{\gamma^2}{16T^2}\left[(2n+1)^2 + 1\right]\left(-\frac{\Delta\epsilon}{k}\right)\left(-\frac{1}{T^2}\right)e^{-\frac{\Delta\epsilon}{kT}}$$

OR

At T_c, $\left(\frac{\partial C_v}{\partial T}\right) = 0$,

$$\left(\frac{\partial C_v}{\partial T}\right)_{T=T_c} = \frac{\gamma^2}{8T_c^3}\frac{\Delta\epsilon}{k}\left[(2n+1)^2 + 1\right]e^{-\frac{\Delta\epsilon}{kT_c}} - \frac{\gamma^2}{16T_c^4}\left[(2n+1)^2 + 1\right]\left(-\frac{\Delta\epsilon}{k}\right)^2 e^{-\frac{\Delta\epsilon}{kT_c}} = 0$$

or

$$T_c = \frac{\Delta\epsilon}{2k} \quad (21)$$

The value of $\Delta\epsilon$ will be obtained from Eq. (18) where $\Delta\epsilon = E_1 - E_0$

For n = 0,

$$E_0 = -4 + \gamma - \frac{\gamma^2}{8} \quad (22)$$

and for n = 1,

$$E_1 = -4 + 3\gamma - \frac{5y^2}{8} \quad (23)$$

Subtracting Eq. (22) from Eq. (23), we get, $\Delta\epsilon$ as,

$$\Delta\epsilon = E_0 - E_1 = -4 + 3\gamma - \frac{5\gamma^2}{8} + 4 - \gamma + \frac{\gamma^2}{8} = 2\gamma - \frac{\gamma^2}{2} \quad (24)$$

for

$$\gamma = 2\pi\Phi$$

And thus

$$T_c = \frac{\Delta\epsilon}{2k} = \pi^2\Phi^2 - 2\pi\Phi \quad (25)$$

Eq. (25) becomes,

$$\pi^2\Phi^2 - 2\pi\Phi - T_c = 0 \qquad (26)$$

Substituting for π in Eq. (26) gives,

$$9.87\Phi^2 - 6.28\Phi - T_c = 0 \qquad (27)$$

OR

$$\Phi = \frac{6.28 \pm \sqrt{39.44 + 39.48T_c}}{19.74} \qquad (28)$$

Eq. (28) is used in studying the magnetic flux per plaquette for any given value of T_c.

5. Results and Discussions

Using equation (6) the plots below in figures 3 and 4 are obtained for H_{c_1} = 100G and H_{c_2} = 3000000G respectively.

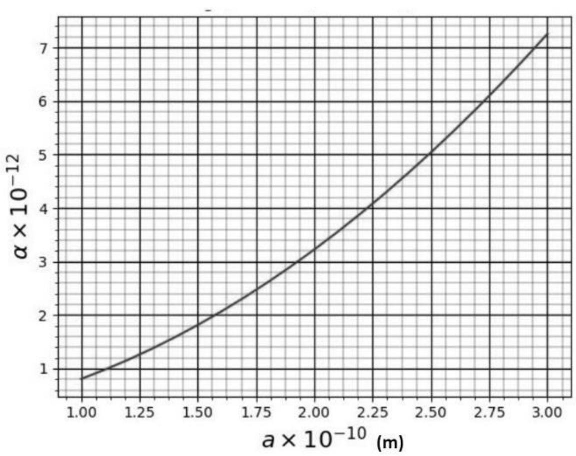

Fig 3. Plot of variation of α with a for H_{c_1} = 100G.

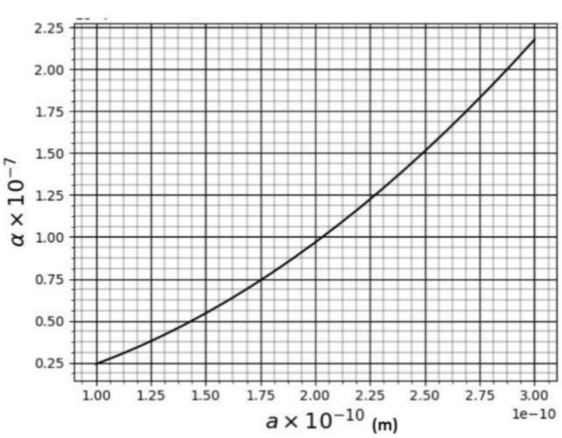

Fig 4. Plot of variation of α with a for H_{c_2} = 3000000G.

For H_{c_1} = 100G the values of α lies between 8.33 x 10^{-13} and 7.5 x 10^{-12} when a = 1Å and when a = 3Å respectively. For YBCO and BSCCO systems and experimental values of H_{c_1} = 100G[20] and a = 1 – 3 Å [21]. For H_{c_2} = 3000000G the values of α lies between 2.5 x 10^{-8} and 2.25 x 10^{-7} when a = 1Å and when a = 3Å respectively. For YBCO and BSCCO systems and experimental values of H_{c_2} = 3000000G[20] and a = 1 – 3 Å[21]. Figures 3 and 4 show that the value of α increases with increase in the applied magnetic field H. Cooper pairs break up at α = 2.25 x 10^{-7}, H_{c_2} = 3000000G and a = 3Å.

Using equation (9) a plot of lattice spacing (a) and transition temperature (T_c) for is obtained as shown in figure 5 below.

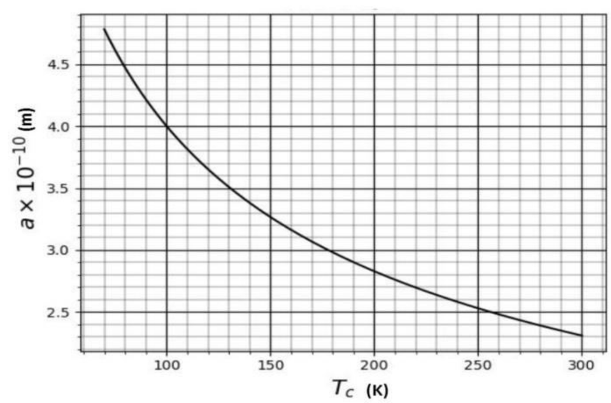

Fig 5. Plot of variation of a with T_c for H_{c_2} = 3000000G.

The plot above shows that there is a non-linear decrease in the value of a with increase in T_c when applied magnetic field is considered. From the graph the value of T_c turns out to be 177K when H_{c_2} = 3000000G for BSSCO and YBCO systems which is almost double the experimental value of T_c = 95K[20]

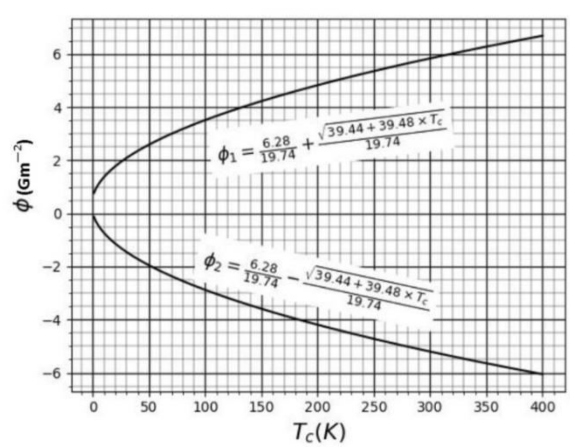

Fig 6. Plot of variation of Φ with T_c.

Using Eq. (28), the variation of Φ against T_c has been plotted as shown in figure 6 above (considering positive valuesof $\Phi = \Phi_1$). This means that for Φ = 5.84 G/m^2, the value of the transition temperature of the superconductor could be close to 300K, which is the room temperature. The value of T_c in the range 77 – 300K may be due to Φ being between 3.12 – 5.84 G/m^2. Thus the increase in Φ leads to increase in T_c or larger flux trapping leads to increase in T_c, emphasizing that the applied magnetic field has a definite role to play in sustaining superconductivity[22,23,24,25].

6. Conclusion

The effect of applied magnetic field on a 2-D square lattice was studied. An electron with its four immediate neighbours undergoes two periodic motions (the period of motion of an electron in a state with a crystal and the characteristic period of the electron under uniform applied magnetic field *H*). The periodic motion of an electron in state with crystal momentum *p* and the periodic time taken by an electron to travel through lattice spacing *a*.The ratio α of the two periodic motions is equal to the ratio of flux through the lattice cell to one flux quantum. The study established that there is no Meissner expulsion of flux as *H* increases through the lattice cell. The value of T_c at which a superconductor changes from superconducting state to normal state and vice versa was found to be 177K for BSCCO and YBCO systems atH_{c_2}= 3.0 x 10^6G, and the value of α is 2.28 x 10^{-7} when H_{c_2}= 3.0 x 10^6G and *a* = 3Å. Also the value of *a* (lattice constant) at T_c = 300K (room temperature) was found to be 2.3Å which is within the experimental range of 1-3Å. Thus a superconductor with lattice constant 2.3Å can be manufactured such that T_c = 300K and H_{c_2}= 3.0 x 10^6G and it is a very important result. The effect of effective magnetic flux per plaquette has also been studied where Harper's equation is subject to second quantization formalism and finally established that there is a linear dependence between the values of $T_c$and effective magnetic flux per plaquette for HTSCs. The value of Φ is 5.84G/m^2 at room temperature (300K) and lies between 3.12-5.84G/m^2 for temperature range of 77-300K.

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