Dynamic Modeling of Surface Mounted Permanent Synchronous Motor for Servo motor application

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ABSTRACT
In this paper the mathematical model of surface mounted permanent magnet synchronous motor (S-PMSM) for servo motor application is presented. Today in many industries the use of Permanent Magnet Synchronous Motor (PMSM) is increasing due to smaller size, less weight & low rotor loss compare to induction motor of same capacity. To make the high efficient operation of the servo device it is necessary that all subsystem must be operating efficiently. The experimental set up imposes a high cost, so the software aided modeling of the system reduces the costs. One of the important characteristics of Permanent Magnet Synchronous Motor is that they produce constant torque with sinusoidal stator current. This paper concentrates on the dynamic modeling and the transient analysis of the PMSM in the normalized unit. The simulation done in MATLAB result shows the good dynamic performance of the PMSM drive.

Keywords *- constant torque, dynamic, modeling, Permanent magnet motor, servo motor.*

I. INTRODUCTION

In industry the PMSM drives are used mostly. The advancement in the permanent materials reduces the cost of the PMSM and makes it more popular in industries. The PMSM drive has a high torque to inertia ratio, good efficiency, low cost, high power density and easy maintenance. The main advantages, as compared with induction motors, are the absence of rotor slip power loss and the natural ability to supply reactive current and this is the main reason that PMSM replace the induction motor drives in industry. Since the magnetic excitation may be provided from the rotor side instead of the stator, the machine can be built with a larger air gap without degraded performance. The ability to supply reactive current also permits the use of natural-commutated dc link converters. These motors also have lower weight, volume, and inertia compared to dc motors for the same ratings.

T. Sebastian [1] reviewed Permanent Magnet Synchronous Motor advancements and presented equivalent electric circuit models for such motors and compared computed parameters with measured parameters. Experimental results on laboratory motors were also given. P. Pillay in [2] presented Permanent Magnet motor drives and classified them into two types such as Permanent Magnet Synchronous Motor drives (PMSM) and Brushless DC Motor (BDCM) drives. The PMSM has a sinusoidal back emf and requires sinusoidal stator currents to produce constant torque while the BDCM has a trapezoidal back emf and requires rectangular stator currents to produce constant torque. B. Cui [3] addresses the modeling and simulation of permanent magnet synchronous motor supplied from a six step continuous inverter based on state space method. The motor model was derived in the stationary reference frame and then in the rotor reference frame using Park transformation. The simulation results obtained showed that the method used for deciding initial conditions was very effective. A brief design review of the Permanent Magnet Synchronous Motors has been presented in [4].A unified lumped parameter circuit model for both steady state and dynamic analysis is developed. The proposed techniques have been experimentally verified in a laboratory permanent magnet synchronous motor. Ilsu Jeong [5] derived a dynamic model for a faulted surface-mount permanent magnet synchronous motor (SPMSM) is using a deformed flux model. In reflecting the internal turn short into the dynamics, the variations in inductance and back EMF term were considered. Abraham Gebregerg in [6] presented a model of permanent-magnet synchronous machine (PMSM), including the electromagnetically originated torque ripple, is presented in this paper. The propose method to quantify the various sources of torque ripple and modifies the existing dq-model to enhance the modeling capabilities for both surface-mount and interior PMSMs. Steady-State Analysis and Comparison of different Control Strategies for PMSM along with merits and limitations are presented in paper [7] which provide a wide variety of control choices in many applications. The performance characteristics for each strategy under steady state are modeled and simulated in MATLAB environment. Based on the comparative study, it can be concluded that constant mutual flux linkages control is

superior and it can be a good control strategy to consider.
In this paper the dynamic modeling of the PMSM is done in MATLAB program. By providing parameters of any specific motor, we get estimated performance characteristics graphs for that particular motor and from these graphs, we easily find out the high efficient operating conditions of the PMSM drives. The results of simulation are discussed. The result obtained validates the advantages of PMSM motor.

II. DYNAMIC MODEL OF PMSM

To drive the dynamic model of the PMSM some assumptions are take the magneto motive force is sinusoidal distributed, the parameter changes are neglected and the system is in balanced condition.

The following steps are followed for the dynamic modeling of the PMSM

Step1: Transform the 3-phase to 2-phase.
Step2: Find out the power equivalence.
Step3: Model the PMSM in rotor reference frame.
Step4: Derive the electromagnetic torque
Step5: Transform the rotor reference frame to stationary
Step6: Derive the per unit or a small signal model
Step7: Find out the frequency response and analysis the stability of the PMSM

The windings are displaced in space by 90^0 electrical degrees and the rotor windings at an angle θ_r from the stator d-axis winding. It is assumed that the q-axis lead the d-axis to a counter clockwise direction of rotation of the rotor [8].

The d-and q-axes stator voltages are derived as the sum of the resistive voltage drops and the derivative of the flux linkages in the respective windings as

$$V_{qs} = R_q i_{qs} + p\lambda_{qs} \tag{1}$$

$$V_{ds} = R_d i_{ds} + p\lambda_{ds} \tag{2}$$

Where-p is the differential operator, d/dt
v_{qs} and $v_{ds}$are the voltage in the q- and d-axes windings
i_{qs} and i_{ds} are the voltage in the q- and d-axes windings
R_q and R_d are the voltage in the q- and d-axes windings
λ_{qs} and $\lambda_{ds}$are the voltage in the q- and d-axes windings

The stator winding flux linkages can be written as the sum of the flux linkages due to their own excitation and mutual flux linkages resulting from other winding current and magnet sources. The q and d stator flux linkages are written as

$$\lambda_{qs} = L_{qq} i_{qs} + L_{qd} i_{ds} + \lambda_{af} \sin\theta_r \tag{3}$$
$$\lambda_{ds} = L_{dq} i_{qs} + L_{qd} i_{ds} + \lambda_{af} \cos\theta_r \tag{4}$$

Where θ_r is the instantaneous rotor position. The windings are balanced and therefore their resistances are equal and denoted as $R_s = R_q = R_d$. The d and q stator voltages can then be written in terms of the flux linkages and resistive voltage drops as

$$V_{qs} = R_s i_{qs} + i_{qs} pL_{qq} + L_{qq} pi_{qs} + L_{qd} pi_{ds} + i_{ds} pL_{qd} + \lambda_{af} p\sin\theta_r \tag{5}$$

$$V_{ds} = R_s i_{ds} + i_{qs} pL_{qd} + L_{qd} pi_{qs} + L_{dd} pi_{ds} + i_{ds} pL_{dd} + \lambda_{af} p\cos\theta_r \tag{6}$$

L_{qq}& L_{dd} = self inductances of the q and d axes windings, respectively

$$L_{qq} = \frac{1}{2}\left[\left(L_q + L_d\right) + \left(L_q - L_d\right)\cos 2\theta_r\right] \tag{7}$$

$$L_{dd} = \frac{1}{2}\left[\left(L_q + L_d\right) - \left(L_q - L_d\right)\cos 2\theta_r\right] \tag{8}$$

Compactly represented as

$$L_{qq} = L_1 + L_2 \cos 2\theta_r \tag{9}$$
$$L_{dd} = L_1 - L_2 \cos 2\theta_r \tag{10}$$

where, L_1&L_2

$$L_1 = \frac{1}{2}\left(L_q + L_d\right) \tag{11}$$

$$L_2 = \frac{1}{2}\left(L_q - L_d\right) \tag{12}$$

$L_{qd} = L_{dq}$= mutual inductances between two windings

$$L_{qd} = -L_2 \sin 2\theta_r \tag{13}$$

Substituting the self and mutual inductances in terms of the rotor position into the stator voltage equations will result in a large number of terms that are rotor position dependent, then the equation becomes:

$$\begin{bmatrix} V_{qs} \\ V_{ds} \end{bmatrix} = R_s \begin{bmatrix} i_{qs} \\ i_{ds} \end{bmatrix} + \begin{bmatrix} L_1 + L_2\cos 2\theta_r & -L_2 \sin 2\theta_r \\ -L_2 \sin 2\theta_r & L_1 - L_2 \cos 2\theta_r \end{bmatrix} \frac{d}{dt} + 2\omega_r L_2 \begin{bmatrix} -\sin\theta_r & -\cos 2\theta_r \\ -\cos 2\,\theta_r & \sin 2\theta_r \end{bmatrix} \begin{bmatrix} i_{qs} \\ i_{ds} \end{bmatrix} + \lambda_{af}\omega_r \begin{bmatrix} \cos\theta_r \\ -\sin\theta_r \end{bmatrix} \tag{14}$$

In surface mount magnet machines, the inductances are equal and therefore L_2 is zero and the above equation becomes for surface mounted PMSM is:

$$\begin{bmatrix} V_{qs} \\ V_{ds} \end{bmatrix} = R_s \begin{bmatrix} i_{qs} \\ i_{ds} \end{bmatrix} + \begin{bmatrix} L_1 & 0 \\ 0 & L_1 \end{bmatrix} \frac{d}{dt} \begin{bmatrix} i_{qs} \\ i_{ds} \end{bmatrix} + \lambda_{af}\omega_r \begin{bmatrix} \cos\theta_r \\ -\sin\theta_r \end{bmatrix} \quad (15)$$

At rotating reference frames, the system inductance matrix becomes independent of the rotor position, thus leading to the simplification and compactness of the system equations. The relationship between the stationary reference frames denoted by d- and q-axes and the rotor reference frames denoted by dr- and qr-axes is written as:

$$i_q d_s = [T^r] i_q^r d_s \quad (16)$$
$$v_q d_s = [T^r] v_q^r d_s \quad (17)$$
$$T^r = \begin{bmatrix} \cos\theta_r & \sin\theta_r \\ -\sin\theta_r & \cos\theta_r \end{bmatrix} \quad (18)$$

$$\begin{bmatrix} v_{qs}^r \\ v_{ds}^r \end{bmatrix} = \begin{bmatrix} R_s + L_q p & \omega_r L_d \\ -\omega_r L_q & R_s + L_d p \end{bmatrix} \begin{bmatrix} i_{qs}^r \\ i_{ds}^r \end{bmatrix} + \begin{bmatrix} \omega_r \lambda_{af} \\ 0 \end{bmatrix} \quad (19)$$

Where ω_r is the rotor speed in electrical radians per second.

$$\begin{bmatrix} i_{qs}^r \\ i_{ds}^r \\ i_0 \end{bmatrix} = \frac{2}{3} \begin{bmatrix} \cos\theta_r & \cos\left(\theta_r - \frac{2\pi}{3}\right) & \cos\left(\theta_r + \frac{2\pi}{3}\right) \\ \sin\theta_r & \sin\left(\theta_r - \frac{2\pi}{3}\right) & \sin\left(\theta_r + \frac{2\pi}{3}\right) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} i_{as} \\ i_{bs} \\ i_{cs} \end{bmatrix} \quad (20)$$

The relationship between the stator currents in the rotor reference frames and the actual stator dq currents is given by

$$i_{qds}^r = [T_r]^{-1} i_{qds} \quad (21)$$

$$[T_{abc}]^{-1} = \begin{bmatrix} \cos\theta_r & \sin\theta_r & 1 \\ \cos\left(\theta_r - \frac{2\pi}{3}\right) & \sin\left(\theta_r - \frac{2\pi}{3}\right) & 1 \\ \cos\left(\theta_r + \frac{2\pi}{3}\right) & \sin\left(\theta_r + \frac{2\pi}{3}\right) & 1 \end{bmatrix} \quad (22)$$

From the modelling, simulation and analysis point of view the power input of the 3-phase and 2-phase machine has to be equal. The input power is

$$p_i = \frac{3}{2}\left[\left(v_{qs}^r i_{qs}^r + v_{ds}^r i_{ds}^r\right) + 2 v_0 i_0\right] \quad (23)$$

In balanced condition the zero sequence current become zero, hence the input power is

$$p_i = \frac{3}{2}\left[\left(v_{qs}^r i_{qs}^r + v_{ds}^r i_{ds}^r\right)\right] \quad (24)$$

The rotor position and speed is determined by the electromagnetic torque. Therefore in simulation study it is important. The electromagnetic torque is derived as:
The dynamic equation of PMSM

$$V = [A]i + [B]pi + [C]\omega_r i \quad (25)$$

Where
[A] matrix consists of resistive elements
[B] matrix consists of the coefficients of the derivative operator p
[C] matrix has elements that are the coefficients of the electrical rotor speed, ω_r

The air gap torque is obtain as
$$\omega_m T_e = P_a \quad (26)$$

$$T_e = \frac{3}{2}\frac{P}{2}\left[\lambda_{af} + \left(L_d - L_q\right) i_{ds}^r\right] i_{qs}^r \quad (\mathrm{N.m}) \quad (27)$$

$$T_e = \frac{3}{2}\frac{P}{2}\left[\lambda_{af} + \left(L_d - L_q\right) I_m \cos\delta\right] I_m \sin\delta \quad (28)$$

$$T_e = \frac{3}{2}\frac{P}{2}\left[\lambda_{af} I_m \sin\delta + \left(L_d - L_q\right) I_m^2 \sin 2\delta\right] \quad (29)$$

$$T_e = T_{es} + T_{er} \quad (30)$$

The air gap torque is the sum of the reluctance and synchronous torque, the peak of the air gap torque is at a torque angle greater than 900 as shown in Fig. 1 the variation of the air gap, synchronous and reluctance torque. The reluctance torque greatly affect the air gap torque it increases the air gap torque in the region 90^0 to 1800 torque angle and decreases the air gap torque when the torque angle lie between 0^0 to 90^0. Therefore, an operation in the first 90^0 is not attempted in these machines and the preferred torque angle is between 90^0 and 180^0.

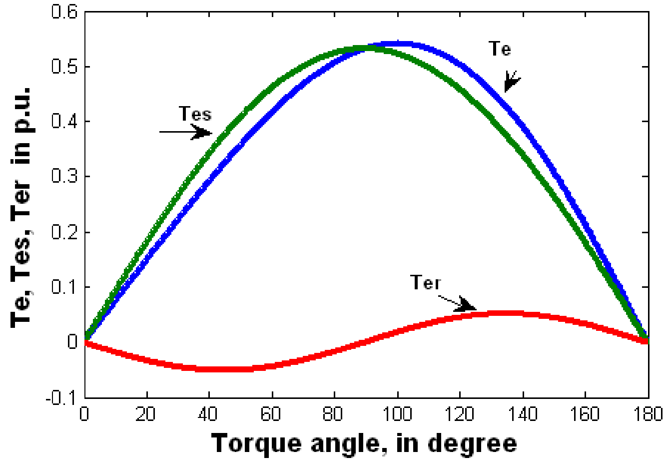

Figure.1 Reluctance, synchronous and air gap torques versus torque angle for stator current magnitude of 1.3 p.u.

From Fig. 2 it is clear that vary the stator current magnitudes changes the angle at which the maximum torque occurs. For the optimal operation of the machine the maximum torque per unit current is one of the performance indices.

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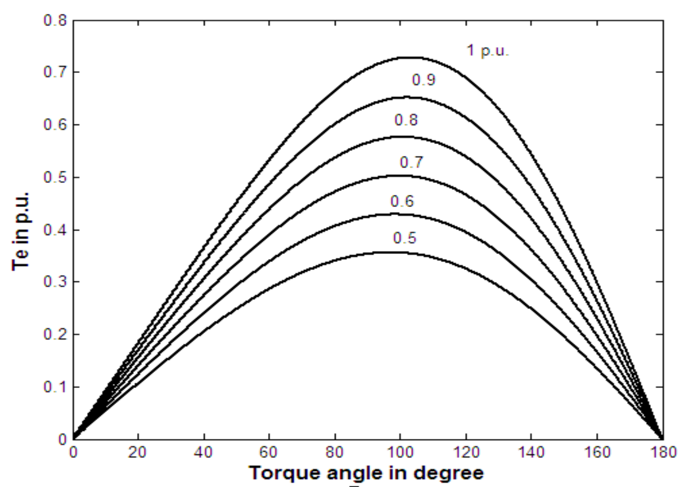

Figure.2 Air gap torque versus torque angle for various stator currents

III. DYNAMIC SIMULATION OF PMSM

The equation of the PMSM in rotor reference frame in normalized units are used for the dynamic simulation these equation facilitates the computer solution. The per unit model of the PMSM is derived by defining the base variables. Let the rms values of the rated phase voltage and current form the base quantities.

The q-axis stator voltage is

$$v_{qs}^{r} = \left(R_s + L_q p\right) i_{qs}^{r} + \omega_r\left(L_d i_{ds}^{r} + \lambda_{af}\right) \quad (31)$$

It is normalized by dividing with the base voltage V_b

$$v_{qs}^{r} = \frac{v_{qs}^{r}}{V_b} = \frac{\left(R_s + L_q p\right)}{V_b} i_{qs}^{r} + \frac{\omega_r\left(L_d i_{ds}^{r} + \lambda_{af}\right)}{V_b} \text{ p.u.} \quad (32)$$

Where, $V_b = I_b Z_b = \omega_b \lambda_b = \omega_b L_b I_b$ (V)

Then,

$$v_{qsn}^{r} = \left(R_{sn} + \frac{L_{qn}}{\omega_b} p\right) i_{qsn}^{r} + \omega_m\left(L_{dn} i_{dsn}^{r} + \lambda_{afn}\right) \text{ p.u.} \quad (33)$$

Similarly, the d-axis stator voltage equation is normalized and given as,

$$v_{dsn}^{r} = -\omega_{rn} L_{qn} i_{dsn}^{r} + \left(R_{sn} + \frac{L_{dn}}{\omega_b} p\right) i_{dsn}^{r} \text{p.u.} \quad (34)$$

The normalized electromagnetic torque is obtained by dividing the torque by base torque. Base torque is define as

$$T_b = \frac{\frac{P_b}{\omega_b}}{P/2} \quad (35)$$

Now,

$$T_{en} = \frac{T_e}{T_b} = 2Hp\omega_{rn} + T_{ln} + B_n \omega_{rn} \quad (36)$$

Where the inertia constant and the normalized friction constant is

$$H = \frac{1}{2} \frac{J\omega_b^2}{P_b\left(P/2\right)^2} \quad \text{(s)} \quad (37)$$

$$B_n = \frac{B\omega_b^2}{P_b\left(P/2\right)^2} \quad (38)$$

And $T_{ln} = \frac{T_l}{T_b}$ the normalized load torque

It is seen that these system equations are nonlinear as, products of variables are involved. Therefore, the only way the system solution can be obtained is by a numerical solution. The solution of the system is then obtained by integrating the differential equations. The optimization method can be used for numerical integration or in the case illustrated in the MATLAB program, a simple solution by discretization can be obtained.

The inputs are q^r- and d^r-axes stator voltages, which are obtained from the abc stator voltages by the transformation of T_{abc}. The transformation requires a rotor position that is obtained at every step of the solution. Then the solution of the equation leads to the solution of the stator currents in the rotor reference frames, speed, and rotor position. The abc phase currents can be obtained from dq currents in rotor reference frames by using the inverse transformation matrix. From the dq stator currents, the electromagnetic torque is obtained for plotting. This is updated for every step of integration until the desired final time is reached in the iteration.

Using the machine parameters given in Table I, the simulation of a direct line-starting of the PMSM is illustrated in the accompanying MATLAB program. The performance under the condition of simulation is shown in Fig. 3

Table: I Machine Parameter

Parameter	Notation	Value
Stator Resistance	R_s	2.4 Ω
d axis inductance	L_d	0.0035 H
q axis inductance	L_q	0.0075 H
Rated speed	ω_r	314.3 rpm
Friction Coefficient	B	0.02 Nms
Rotor Flux linkage	λ_{af}	0.1845 Wb
Moment of Inertia	J	0.006 kg/m2
Number of Poles	p	6

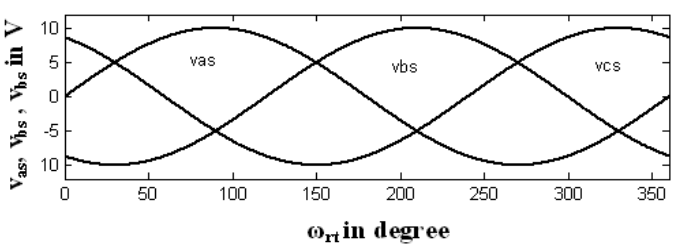

Fig.3(a)

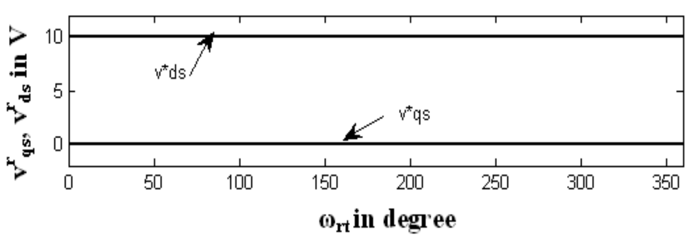

Fig.3(b)

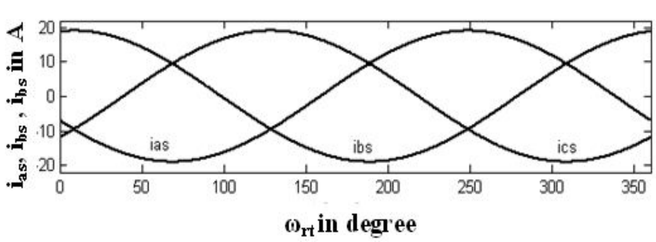

Fig.3(c)

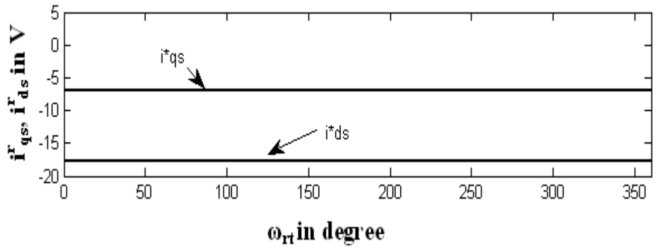

Fig.3(d)

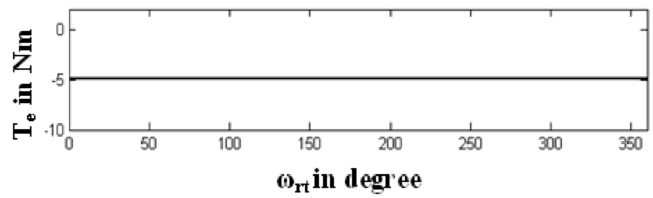

Fig.3(e)

Figure.3 The rotor reference frame and abc frames based stator voltages and currents and electromagnetic torque at constant speed.

For the simulation, the load torque is considered to be zero and the applied phase voltages are equal to the base voltage in magnitude and a set of balanced three-phase voltages is impressed at 50 Hz. The q- and d-axes stator voltages in the rotor reference frames are not constants but oscillatory as shown in figure 3(b). It is because the rotor position derived from rotor speed is oscillatory. The oscillatory rotor position influences the stator currents in rotor reference frames because the transformation matrix is solely a function of rotor position.

There is no control imposed on the PMSM based on its rotor position in this simulation. Because of this, the stator currents attain high values with the attendant oscillation in air gap torque, resulting in a significant oscillation of the rotor as shown in figure 3(d).Such an operation is undesirable. A feedback control signal is used to stabilize the signals.

IV. CONCLUSION

The PMSM drives are used as servo motor application for increasing the performance of the drive it should be ensure that PMSM operate at the optimum mode. In this paper the dynamic modeling of the PMSM has been presented that should be help to model the PMSM controller.

In a PMSM where inductances vary as a function of rotor angle, the 2 phase (d-q) equivalent circuit model is basically used for reduce the complexity. In this paper, a two phase model for a PM synchronous motor is derived and relate with the 3-phase. Modeling is done mainly for surface permanent magnet motors but can also be applied to interior permanent magnet synchronous motors.

V. ACKNOWLEDGEMENT

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