SOLVING FUZZY LINEAR PROGRAMMING PROBLEM USING SUPPORT AND CORE OF FUZZY NUMBERS

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ABSTRACT
Many authors used different types ranking function to solve fuzzy linear programming method. In this paper new approach has been proposed to solve fuzzy linear programmed by using support and core of fuzzy numbers without using membership function and alpha cut technique. Different types of problems were taken and solved by proposed method.

Keywords *– Fuzzy linear programming problem, Trapezoidal and Triangular fuzzy number, support and core of fuzzy number.*

1. INTRODUCTION

The concept of decision-making in fuzzy environment first proposed by R.E.Bellman and L.A Zadah [1].
Zimmerann[2] has proposed the concept of fuzzy linear programming. Many authors proposed a different technique of solving FLP problem in real time situation. Maleki [4] has proposed a new method of solving FLP problem using ranking functions. P.Rajarajeshwari and Sahaya Sudha [8] proposed a method to solve a FFLP problem by Robust's ranking function and the compared their solution with Pandian and Jayalakshmi [5] solution who proposed a new method to solve FFLP problem.Later on Iden Hassan Alkanani and Farrah Alaa Adnan [9] has used Maleki [4] and Yaker ranking functions to solve FLP when fuzzy numbers in objective function coefficients, fuzzy numbers in right-hand side coefficients and finally fuzzy numbers in objective function coefficients as well as in right-hand side coffeicients.A.Kumar and Singh[7] proposed a new method for solving fully fuzzy linear programming problems using ranking function.

2. DEFINITIONS

2.1. Membership function: Let R be the real line. Let A be the subset of R. The function $\mu_A : R \to [0,1]$ is known as membership function on X.

Fuzzy set: Let X be set and x be an arbitrary element of X, then a fuzzy subset A of X is a map $\mu_A : X \to [0,1]$ (or) is a set of ordered pair $A = \{(x, \mu_A(x)) \mid x \in X\}$, where $\mu_A(x)$ is a membership function.

2.2 Fuzzy number: A fuzzy set A of R is said to be trapezoidal fuzzy number , if the membership function has the following characteristic

1. $\mu_A : R \to [0,1]$ is continuous function.
2. $\mu_A(x) = 0, \forall x \in (-\infty, c] \cup [d, \infty)$.
3. Strictly increasing on [c,a] and strictly decreasing on [b,d].
4. $\mu_A(x) = 1, \forall x \in [a,b]$

Note: Trapezoidal Fuzzy number becomes triangular fuzzy number when a=b.

2.3 α cut: The α cut of a fuzzy set A is the crisp set , A_α ,that contains all the elements of X whose membership grade in A are greater than or equal to the value α .

i.e., $A_\alpha = \{x \mid \mu_A(x) \ge \alpha\}$

2.4 Convex fuzzy set: A fuzzy set A on R is convex if and only if $\mu_A(\lambda x + (1-\lambda) y) \ge \min(\mu_A(x), \mu_A(y))$, $\forall x, y \in X$ and $\lambda \in [0,1]$

2.5 Support of Fuzzy number: If A is a fuzzy set, then support of $A = \{x \in X \mid \mu_A(x) > 0\}$ and denoted by Supp(A) or S(A).

2.6. Core of fuzzy number: If A is a fuzzy set, then core of $A = \{x \in X \mid \mu_A(x) = 1\}$ and denoted by Cor(A) or C(A).

2.7. Average of an interval: If [a,b] is a subset of R, then average of [a, b] is defined as Avg([a,b])= $\frac{a+b}{2}$.

There are many ways of representing the fuzzy numbers. But most importantly the fuzzy numbers are represented in two ways namely triangular fuzzy number and trapezoidal fuzzy numbers.

2.8.Trapezoidal fuzzy number:

Let us consider the trapezoidal number as
$A=(a_l,a_u,\alpha,\beta)=(a_l-\alpha,a_l,a_u,a_u+\beta)$

where $\alpha=\beta$ or $\alpha\neq\beta$ and its corresponding membership function is defined as follows

$$\mu_A(x)=\begin{cases}\frac{x-(a_l-\alpha)}{\alpha} & a_l-\alpha\le x\le a_l\\ 1 & a_l\le x\le a_u\\ \frac{(a_u+\beta)-x}{\beta} & a_u\le x\le a_u+\beta\\ 0 & elsewhere\end{cases}$$

The geometrical representation of trapezoidal membership function for fuzzy number (a_l,a_u,α,β) is shown below

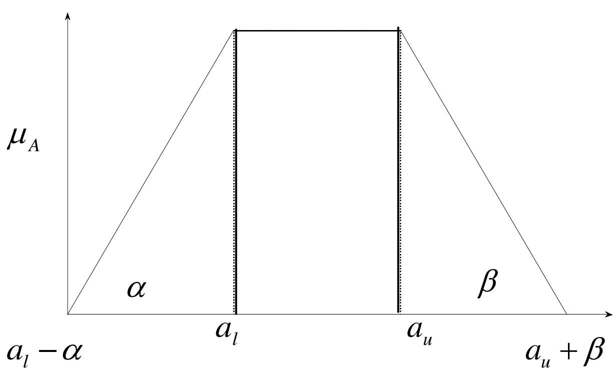

Fig-1: Trapezoidal Fuzzy Number

Here, the support of A is $S(A)=(a_l-\alpha,a_u+\beta)$ and the core of A is $C(A)=[a_l,a_u]$.

$$Avg(S(A))=\frac{(a_l-\alpha)+(a_u+\beta)}{2}$$ and

$$Avg(C(A))=\frac{(a_l+a_u)}{2}\;\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

2.9.Triangular fuzzy number:

Let us consider the trapezoidal number as
$A=(a,\alpha,\beta)=(a-\alpha,a,a+\beta)$ where $\alpha=\beta$ or $\alpha\neq\beta$ and its corresponding membership function is defined as follows

$$\mu_A(x)=\begin{cases}\frac{x-(a-\alpha)}{\alpha} & a-\alpha\le x\le a\\ 1 & x=a\\ \frac{(a+\beta)-x}{\beta} & a\le x\le a+\beta\\ 0 & elsewhere\end{cases}$$

The geometrical representation of trapezoidal membership function for fuzzy number (a,α,β) is shown below

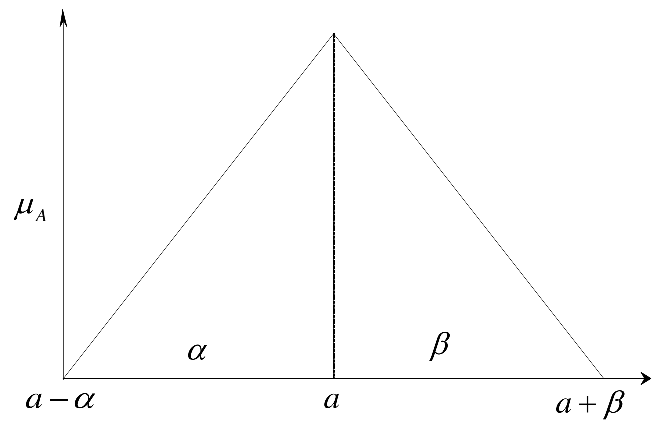

Fig-2: Triangular Fuzzy Number

Here, the support of A is $S(A)=(a-\alpha,a+\beta)$ and the core of A is $C(A)=\{a\}$.

$$Avg(S(A))=\frac{(a-\alpha)+(a+\beta)}{2},$$

$$Avg(C(A))=\frac{a}{2}\;\ldots\ldots\ldots\ldots\ldots\ldots..(2)$$

2.10. Operations of Trapezoidal Fuzzy Numbers:-

Assume that $A=(a_l,a_u,\alpha,\beta)$ and $B=(b_l,b_u,\gamma,\delta)$ are any two trapezoidal fuzzy number , then the arithmetic trapezoidal fuzzy numbers are defined as below

2.10.1. Addition: $A+B=(a_l,a_u,\alpha,\beta)+(b_l,b_u,\gamma,\delta)$
$=(a_l+b_l,a_u+b_u,\alpha+\gamma,\beta+\delta)$

2.10.2. Subtraction: $A-B=(a_l,a_u,\alpha,\beta)-(b_l,b_u,\gamma,\delta)$
$=(a_l-b_u,a_u-b_l,\alpha+\delta,\beta+\gamma)$

2.10.3.Scalar Multiplication:
$xA=(xa_l,xa_u,x\alpha,x\beta),\ if\ \text{x}\geq 0$ and
$xA=(xa_u,xa_l,-x\beta,-x\alpha),\ if\ \text{x}<0$

2.11.Operations of Triangular Fuzzy Numbers:-

Assume that $A=(a,\alpha,\beta)$ and $B=(b,\gamma,\delta)$ are any two trapezoidal fuzzy number , then the arithmetic trapezoidal fuzzy numbers are defined as below

2.11.1.Addition: $A+B=(a,\alpha,\beta)+(b,\gamma,\delta)$
$=(a+b,\alpha+\gamma,\beta+\delta)$

2.11.2.Subtraction: $A-B=(a,\alpha,\beta)-(b,\gamma,\delta)$
$=(a-b,\alpha+\delta,\beta+\gamma)$

2.11.3.Scalar Multiplication:
$xA=(xa,x\alpha,x\beta),\ if\ \text{x}\geq 0$ and
$xA=(xa,-x\beta,-x\alpha),\ if\ \text{x}<0$

2.12.Ranking function: The ranking function is the most powerful technique to compare the fuzzy numbers. Many types of ranking functions introduced by the many authors to solve fuzzy linear programming problem with fuzzy parameters.

Let F(R) be the set of all fuzzy numbers. The ranking function $\Re$ is defined by $\Re: F(R)\to R$.

The comparison between the fuzzy numbers A and B is shown below

1. $A<B$ iff $\Re(A)<\Re(B)$
2. $A>B$ iff $\Re(A)>\Re(B)$
3. $A=B$ iff $\Re(A)=\Re(B)$

Also $\Re$ satisfies the linear property as follows $\Re(cA+B)=c\Re(A)+\Re(B)$,where $c\in R$ and $A,B\in F(R)$.

2.13. Support and core with respect to Maleki Ranking Function for Trapezoidal fuzzy numbers:-

If $A=(a_l,a_u,\alpha,\beta)=(a_l-\alpha,a_l,a_u,a_u+\beta)$ is a trapezoidal fuzzy number, then the Maleki ranking function[4] for trapezoidal fuzzy number is

$$\Re(A)=\int_0^1(\inf a_\lambda+\sup a_\lambda)d\lambda=a_l+a_u+\frac{(\beta-\alpha)}{2}$$

$$=\frac{2(a_l+a_u)+(\beta-\alpha)}{2}$$

$$=\frac{(a_l+a_u)+(\beta-\alpha)+(a_l+a_u)}{2}$$

$$=\frac{(a_l-\alpha)+(a_u+\beta)+(a_l+a_u)}{2}$$

$$=\frac{(a_l-\alpha)+(a_u+\beta)}{2}+\frac{(a_l+a_u)}{2}$$

$$\Re(A)=Avg(S(A))+Avg(C(A))$$

2.14.Support and core with respect to Maleki Ranking Function for Triangular fuzzy numbers:-

If $A=(a,\alpha,\beta)=(a-\alpha,a,a+\beta)$ is a triangular fuzzy number, then the Maleki ranking function[4] for triangular fuzzy number is

$$\Re(A)=\int_0^1(\inf a_\lambda+\sup a_\lambda)d\lambda=2a+\frac{(\beta-\alpha)}{2}$$

$$=\frac{4a+(\beta-\alpha)}{2}=\frac{(a-\alpha)+(a+\beta)+2a}{2}$$

$$=\frac{(a-\alpha)+(a+\beta)}{2}+a$$

$$\Re(A)=Avg(S(A))+C(A)$$

2.15.Support and core with respect to Robust's Ranking Function for Trapezoidal fuzzy numbers:-

$$\Re(A)=\frac{1}{2}\int_0^1(\inf a_\lambda+\sup a_\lambda)d\lambda$$
$$=\frac{1}{2}\left(a_l+a_u+\frac{(\beta-\alpha)}{2}\right)$$
$$=\frac{1}{2}\left(\frac{2(a_l+a_u)+(\beta-\alpha)}{2}\right)$$
$$=\frac{1}{2}\left(\frac{(a_l+a_u)+(\beta-\alpha)+(a_l+a_u)}{2}\right)$$
$$=\frac{1}{2}\left(\frac{(a_l-\alpha)+(a_u+\beta)+(a_l+a_u)}{2}\right)$$
$$=\frac{1}{2}\left(\frac{(a_l-\alpha)+(a_u+\beta)}{2}+\frac{(a_l+a_u)}{2}\right)$$
$$\Re(A)=\frac{1}{2}(Avg(S(A))+Avg(C(A)))$$

2.16.Support and core with respect to Robust's Ranking Function for Triangular fuzzy numbers:-

If $A=(a,\alpha,\beta)=(a-\alpha,a,a+\beta)$ is a triangular fuzzy number, then the Robust's ranking function for triangular fuzzy number is

$$\Re(A)=\frac{1}{2}\int_0^1(\inf a_\lambda+\sup a_\lambda)d\lambda=\frac{1}{2}\left(2a+\frac{(\beta-\alpha)}{2}\right)$$
$$=\frac{1}{2}\left(\frac{4a+(\beta-\alpha)}{2}\right)$$
$$=\frac{1}{2}\left(\frac{(a-\alpha)+(a+\beta)+2a}{2}\right)$$
$$=\frac{1}{2}\left(\frac{(a-\alpha)+(a+\beta)}{2}+a\right)$$
$$\Re(A)=\frac{1}{2}(Avg(S(A))+C(A))$$

2.16.Mathematical formulation of Linear programming Problem:

The mathematical linear programming problem is stated as follows

$$Max\text{ (or) }\mathrm{Min}\ z=\sum_{j=1}^{n}c_jx_j \text{ subject to}$$
$$\sum_{j=1}^{n}a_{ij}x_j(\le or = or \ge)b_j\text{ , i=1,2........m}$$
$$x_j\ge 0$$

The above mathematical LPP is known as crisp linear programming problem. Here all the parameters are crisp values. Some time the some or all the parameters are fuzzy numbers, then the crisp LPP became fuzzy linear programming problem.

So, the mathematical fuzzy linear programming problem as follows

$$Max\text{ (or) }\mathrm{Min}\ z=\sum_{j=1}^{n}\tilde{c}_jx_j \text{ subject to}$$
$$\sum_{j=1}^{n}\tilde{a}_{ij}x_j(\le or = or \ge)\tilde{b}_j\text{ , i=1,2......m}$$
$$x_j\ge 0$$

Where $\tilde{c}_j$, $\tilde{a}_{ij}$, $\tilde{b}_j$ are fuzzy numbers.

Numerical Examples:

1. For symmetric trapezoidal fuzzy number

$$Max\ \tilde{z}=(4,8,2,2)\tilde{x}_1+(1,3,1,1)\tilde{x}_2+(1,5,2,2)\tilde{x}_3$$
subject to
$$2\tilde{x}_1-\tilde{x}_2+2\tilde{x}_3\le(1,7,3,3)$$
$$\tilde{x}_1+4\tilde{x}_3\le(2,6,2,2)$$
$$\tilde{x}_1+3\tilde{x}_2+2\tilde{x}_3\le(5,9,2,2)\ ,\ \tilde{x}_1,\ \tilde{x}_2,\ \tilde{x}_3\ge 0$$

Using Maleki Ranking function,
Support, core and ranking function value for the fuzzy number A=(4,8,2,2)calculated as bellow

$$S(A)=(a_l-\alpha,a_u+\beta)=(4-2,8+2)=(2,10)$$
$$Avg(S(A))=\frac{(a_l-\alpha)+(a_u+\beta)}{2}=\frac{2+10}{2}=\frac{12}{2}=6$$
$$C(A)=[a_l,a_u]=[4,8]$$

$$Avg(C(A)) = \frac{(a_l + a_u)}{2} = \frac{(4+8)}{2} = \frac{12}{2} = 6$$

Maleki Ranking function value for =(4,8,2,2) is $\Re(A) = Avg(S(A)) + AvgC(A) = 6 + 6 = 12$

Similar manner crisp values for the remaining fuzzy number found and list in the table below

Fuzzy number(A)	S(A)	C(A)	Avg (S(A))	Avg (C(A))	$\Re(A)$
(1,3,1,1)	(0,4)	[1,3]	2	2	4
(1,5,2,2)	(-1,6)	[1,5]	3	3	6
(1,7,3,3)	(-2,10)	[1,7]	4	4	8
(2,6,2,2)	(0,8)	[2,6]	4	4	8
(5,9,2,2)	(3,11)	[5,9]	7	7	14

So, the crisp LPP for the above FLPP as given below

$$Max\ z = 12x_1 + 4x_2 + 6x_3 \text{ subject to}$$
$$2x_1 - x_2 + 2x_3 \le 8$$
$$x_1 + 4x_3 \le 8$$
$$x_1 + 3x_2 + 2x_3 \le 14 \quad x_1,\ x_2,\ x_3 \ge 0$$

Where z, x_1, x_2, x_3 are crisp variables corresponding to fuzzy variables $\tilde{z}$, $\tilde{x}_1$, $\tilde{x}_2$, $\tilde{x}_3$

Solving the above LPP using simplex method we get,
x_1=5.428, x_2=2.857, x_3=0 and z=76.57

Using Robust's Ranking function,
Support, core and ranking function value for the fuzzy number A=(4,8,2,2)calculated as bellow

$$S(A) = (a_l - \alpha, a_u + \beta) = (4-2, 8+2) = (2,10)$$

$$Avg(S(A)) = \frac{(a_l - \alpha) + (a_u + \beta)}{2} = \frac{2+10}{2} = \frac{12}{2} = 6$$

$$C(A) = [a_l, a_u] = [4,8]$$

$$Avg(C(A)) = \frac{(a_l + a_u)}{2} = \frac{(4+8)}{2} = \frac{12}{2} = 6$$

Robust's Ranking function value for =(4,8,2,2 is $\Re(A) = \frac{1}{2}\left(Avg(S(A)) + AvgC(A)\right) = \frac{1}{2}(6+6) = 6$

Similar manner crisp values for the remaining fuzzy number found and list in the table below

Fuzzy number(A)	S(A)	C(A)	Avg (S(A))	Avg (C(A))	$\Re(A)$
(1,3,1,1)	(0,4)	[1,3]	2	2	2
(1,5,2,2)	(-1,6)	[1,5]	3	3	3
(1,7,3,3)	(-2,10)	[1,7]	4	4	4
(2,6,2,2)	(0,8)	[2,6]	4	4	4
(5,9,2,2)	(3,11)	[5,9]	7	7	7

So, the crisp LPP for the above FLPP as given below

$$Max\ z = 6x_1 + 2x_2 + 3x_3 \text{ subject to}$$
$$2x_1 - x_2 + 2x_3 \le 4$$
$$x_1 + 4x_3 \le 4$$
$$x_1 + 3x_2 + 2x_3 \le 7$$
$$x_1,\ x_2,\ x_3 \ge 0$$

Where z, x_1, x_2, x_3 are crisp variables corresponding to fuzzy variables $\tilde{z}$, $\tilde{x}_1$, $\tilde{x}_2$, $\tilde{x}_3$

Solving the above LPP using simplex method we get,
x_1= 2.7143, x_2= 1.4286, x_3=0 and z= 19.1429

2. For non-symmetric trapezoidal fuzzy number

$$Max\ \tilde{z} = (4,8,3,1)\tilde{x}_1 + (1,3,1,1)\tilde{x}_2 + (1,5,3,1)\tilde{x}_3$$
subject to
$$2\tilde{x}_1 - \tilde{x}_2 + 2\tilde{x}_3 \le (1,7,4,2)$$
$$\tilde{x}_1 + 4\tilde{x}_3 \le (2,6,1,3)$$
$$\tilde{x}_1 + 3\tilde{x}_2 + 2\tilde{x}_3 \le (5,9,1,3),\quad \tilde{x}_1,\ \tilde{x}_2,\ \tilde{x}_3 \ge 0$$

Using Maleki Ranking function,
Support, core and ranking function value for the fuzzy number A=(4,8,3,1)calculated as bellow

$$S(A) = (a_l - \alpha, a_u + \beta) = (4-3, 8+1) = (1,9)$$

$$Avg(S(A)) = \frac{(a_l - \alpha) + (a_u + \beta)}{2} = \frac{1+9}{2} = \frac{10}{2} = 5$$

$$C(A) = [a_l, a_u] = [4,8]$$

$$Avg(C(A)) = \frac{(a_l + a_u)}{2} = \frac{(4+8)}{2} = \frac{12}{2} = 6$$

$$\Re(A) = Avg(S(A)) + AvgC(A) = 5 + 6 = 11$$

Similar manner crisp values for the remaining fuzzy number found and list in the table below

Fuzzy number(A)	S(A)	C(A)	Avg (S(A))	Avg (C(A))	$\Re(A)$
(1,3,1,1)	(0,4)	[1,3]	2	2	4
(1,5,3,1)	(-2,6)	[1,5]	2	3	5
(1,7,4,2)	(-3,9)	[1,7]	3	4	7
(2,6,1,3)	(1,9)	[2,6]	5	4	9
(5,9,1,3)	(4,12)	[5,9]	8	7	15

So, the crisp LPP for the above FLPP as given below

$$Max\ z = 11x_1 + 4x_2 + 5x_3 \text{ subject to}$$
$$2x_1 - x_2 + 2x_3 \le 7$$
$$x_1 + 4x_3 \le 9$$
$$x_1 + 3x_2 + 2x_3 \le 15,\quad x_1,\ x_2,\ x_3 \ge 0$$

Where x_1, x_2, x_3 are crisp variables corresponding to fuzzy variables $\tilde{x}_1$, $\tilde{x}_2$, $\tilde{x}_3$

Solving the above LPP using simplex method we get, x_1=5.143, x_2=3.256, x_3=0 and z=69.714

Using Robust's Ranking function,
Support, core and ranking function value for the fuzzy number A=(4,8,3,1)calculated as bellow

$$S(A) = (a_l - \alpha, a_u + \beta) = (4-3, 8+1) = (1,9)$$

$$Avg(S(A)) = \frac{(a_l - \alpha) + (a_u + \beta)}{2} = \frac{1+9}{2} = \frac{10}{2} = 5$$

$$C(A) = [a_l, a_u] = [4,8]$$

$$Avg(C(A)) = \frac{(a_l + a_u)}{2} = \frac{(4+8)}{2} = \frac{12}{2} = 6$$

$$\Re(A) = \frac{1}{2}\left(Avg(S(A)) + AvgC(A)\right) = \frac{1}{2}(5+6) = 5.5$$

Similar manner crisp values for the remaining fuzzy number found and list in the table below

Fuzzy number(A)	S(A)	C(A)	Avg (S(A))	Avg (C(A))	$\Re(A)$
(1,3,1,1)	(0,4)	[1,3]	2	2	2
(1,5,3,1)	(-2,6)	[1,5]	2	3	2.5
(1,7,4,2)	(-3,9)	[1,7]	3	4	3.5
(2,6,1,3)	(1,9)	[2,6]	5	4	4.5
(5,9,1,3)	(4,12)	[5,9]	8	7	7.5

So, the crisp LPP for the above FLPP as given below

$$Max\ z = 5.5x_1 + 2x_2 + 2.5x_3 \text{ subject to}$$
$$2x_1 - x_2 + 2x_3 \le 3.5$$
$$x_1 + 4x_3 \le 4.5$$
$$x_1 + 3x_2 + 2x_3 \le 7.5,\quad x_1,\ x_2,\ x_3 \ge 0$$

Where x_1, x_2, x_3 are crisp variables corresponding to fuzzy variables $\tilde{x}_1$, $\tilde{x}_2$, $\tilde{x}_3$

Solving the above LPP using simplex method we get, x_1= 2.5714, x_2= 1.6429, x_3=0 and z= 17.4286

Now, consider triangular Fully Fuzzy linear programming problem the problem discussed in Rajarajeshwari[8]

$$Max\ \tilde{z} \approx (1,2,3) \otimes \tilde{x}_1 + (2,3,4) \otimes \tilde{x}_2 \text{ subject to}$$
$$(0,1,2) \otimes \tilde{x}_1 + (1,2,3) \otimes \tilde{x}_2 \approx (1,10,27)$$
$$(1,2,3) \otimes \tilde{x}_1 + (0,1,2) \otimes \tilde{x}_2 \approx (2,11,28)$$
$$\tilde{x}_1,\ \tilde{x}_2 \ge 0$$

Solution:-

By Maleki ranking function,we get the crisp LPP as follows

$$Max\ z = 4x_1 + 6x_2 \text{ subject to}$$
$$2x_1 + 4x_2 \le 24$$
$$4x_1 + 2x_2 \le 26,\quad x_1,\ x_2 \ge 0$$

The solution is x_1= 4.6667; x_2 =3.6667; Z= 40.6667

By Robust's ranking function, we get the crisp LPP as follows

$$Max\ z = 2x_1 + 3x_2 \text{ subject to}$$
$$x_1 + 2x_2 \le 12$$
$$2x_1 + x_2 \le 13,\quad x_1,\ x_2 \ge 0$$

The solution is x_1= 4.6667; x_2 =3.6667; Z= 20.3333

3. CONCLUSION

In this paper we proposed new method of solving any kind Fuzzy linear programming problem without using alpha cut method. Here we introduced the new technique support and core of trapezoidal and triangular fuzzy numbers with different types of problem. When compare with other proposed methods by various authors this will be the simplest method of solving any FLPP. We have taken Maleki ranking function and Robust's Ranking function for solving FLPP.

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