Solving Fully Fuzzy Linear Systems with Triangular Fuzzy Number Matrices by Partitioning

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ABSTRACT

In this paper, a fully fuzzy linear system is solved by partitioning the coefficient matrix into sub matrices with triangular fuzzy number matrices. The method is illustrated with numerical examples.

Keywords *- Fully Fuzzy Linear Systems(FFLS), Triangular fuzzy number matrices, partitioning, Schur complement*

I. INTRODUCTION

Linear system of equation has applications in many areas of science, engineering, finance and economics. Fuzzy linear system whose coefficient matrix is crisp and right hand side column is an arbitrary fuzzy number was first proposed by Friedman et al [4].

A linear system is called a fully fuzzy linear system (FFLS) if all coefficients in the system are all fuzzy numbers .Several direct and iterative methods based on numerical algorithms were used for solving fuzzy linear systems have been introduced by many authors [1,2,3,8].

In this paper a FFLS is solved by partitioning the coefficient matrix into sub matrices with triangular fuzzy numbers matrices. This paper is organized as follows. Some basic definitions and results on fuzzy sets and triangular fuzzy numbers are given in section 2.In section 3, method to solve FFLS with triangular fuzzy numbers matrices by partitioning is given. Illustrations with numerical examples are given in section 4. Section 5 ends this paper with conclusion.

II. PRELIMINARIES

1. A fuzzy set is characterized by a membership function mapping the element of a domain, space or universe of discourse X to the unit interval [0,1]. A fuzzy set A in a universe of discourse X is defined as the following set of pairs

$$A = \{(x, \mu_A(x))\ ;\ x \in X\}$$

Here $\mu_A : X \to [0,1]$ is a mapping called the degree of membership function of the fuzzy set A and μ_A (x) is called the membership value of $x \in X$ on the fuzzy set A. These membership grades are often represented by real numbers ranging from [0,1].

2. A triangular fuzzy number denoted by M = (m,, α, β) has the membership function

$$\mu_{\tilde{A}_{LR}}(\mathrm{x}) = \begin{cases} 0 & ; -\infty < x \le \mathrm{m\alpha} \\ 1 - \dfrac{\mathrm{m-x}}{\alpha} & ; \mathrm{m - d \le x} < m \\ 1 - \dfrac{\mathrm{x-m}}{\beta} & ; \mathrm{m \le x} < x + \beta \\ 0 & ; \mathrm{m} + \beta \le \mathrm{x} < \infty \end{cases}$$

The basic operations on TFNs. Here we introduce the definition of arithmetic operation.

Let M= (m, α, β) and N = (x,y,γ) be two TFNs.

3. A triangular fuzzy number $\tilde{A} = (m, \alpha, \beta)$ is said to be zero triangular fuzzy number if and only if $m = 0, \alpha = 0, \beta = 0.$
4. Two fuzzy number $M = (m, \alpha, \beta)$ and $N = (x, \gamma, \delta)$ are equal if and only if $m = x, \alpha = \gamma, \beta = \delta.$
5. For two fuzzy numbers $M = (m, \alpha, \beta)$ and $N = (x, \gamma, \delta)$ the operations extended addition, extended opposite and extended multiplication are

$$(m, \alpha, \beta) \oplus (x, \gamma, \delta) = (m + x, \alpha + \gamma, \beta + \delta)$$

$$-M = -(m, \alpha, \beta) = (-m, \alpha, \beta)$$

For scalar multiplication

$$\lambda \otimes (m, \alpha, \beta) = \begin{cases} \lambda m, \lambda\alpha, \lambda\beta & \lambda \geq 0 \\ \lambda m, -\lambda\alpha, -\lambda\beta & \lambda < 0 \end{cases}$$

6. A matrix $\tilde{A} = (\tilde{a}_{ij})$ is called a fuzzy matrix if each element of $\tilde{A}$ is a fuzzy number. A fuzzy matrix $\tilde{A}$ is positive denoted by $\tilde{A} > 0$ if each element of $\tilde{A}$ is positive. Fuzzy matrix $\tilde{A} = (\tilde{a}_{ij})$ which is $n \times n$ matrix can be represented such that $\tilde{a}_{ij} = (a_{ij}, b_{ij} m_{ij})$ where $\tilde{A} = (A, B, M)$ $A = (a_{ij}), B = (b_{ij}), M = (m_{ij})$ are $n \times n$ crisp matrices.
7. A square matrix $\tilde{A} = (\tilde{a}_{ij})$ is symmetric matrix $\tilde{a}_{ij} = \tilde{a}_{ji} for\ all\ ij$
8. Consider $n \times n$ fuzzy linear system of equations

$$(\tilde{a}_{11} \otimes \tilde{x}_1) \oplus (\tilde{a}_{12} \otimes \tilde{x}_2) \oplus \ldots \oplus (\tilde{a}_{1n} \otimes \tilde{x}_n) = \tilde{b}_1.$$

$$(\tilde{a}_{21} \otimes \tilde{x}_1) \oplus (\tilde{a}_{22} \otimes \tilde{x}_2) \oplus \ldots \oplus (\tilde{a}_{2n} \otimes \tilde{x}_n) = \tilde{b}_2.$$

....

$$(\tilde{a}_{n1} \otimes \tilde{x}_1) \oplus (\tilde{a}_{n2} \otimes \tilde{x}_2) \oplus \ldots \oplus (\tilde{a}_{nn} \otimes \tilde{x}_n) = \tilde{b}_n.$$

The matrix of the above equation is $\tilde{A} \otimes \tilde{x} = \tilde{b}$ where coefficient matrix $\tilde{A} = (\tilde{a}_{ij})$ where $1 \leq i.j \leq n$ is a $n \times n$ fuzzy matrix and $\tilde{x}_j, \tilde{b}_j \in F(R)$. this system is called fully fuzzy linear system.

III. SOLVING FULLY FUZZY LINEAR SYSTEM USING TRIANGULAR FUZZY NUMBER MATRICES BY PARTITIONING.

If in a given fully fuzzy linear system $\tilde{A} \otimes \tilde{x} = \tilde{b}$…(1) the order of $\tilde{A}$ is very large then computing inverse becomes very difficult .The given FFLS is converted to equivalent crisp system , which is partitioned into sub matrices using Schur complement . It is easier to work with these sub matrices as their orders are considerably reduced.

For solving $n \times n$ fully fuzzy linear system $\tilde{A} \otimes \tilde{x} = \tilde{b}$ where

$$\tilde{A} = (A, B, M), \tilde{x} = (x, y, z) \text{ and } \tilde{b} = (b, g, h)$$

$$(A, B, M) \otimes (x, y, z) = (b, g, h). \quad using\ (1)$$

$$(Ax, By, Az + Mx) = (b, g, h)$$

$$Ax = b \text{.......(3.1)}$$

$By = g$…….(3.2)

$Ax + Mx = h$……..(3.3)

The crisp linear system $Ax = b$ and $By = g$ can be partitioned into sub matrices using schur component such that

$$\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} x^{(1)} \\ x^{(2)} \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} \text{………(4.1)}$$

$$\begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} \begin{bmatrix} y^{(1)} \\ y^{(2)} \end{bmatrix} = \begin{bmatrix} g_1 \\ g_2 \end{bmatrix} \text{………(4.2)}$$

Here each A_{ij} is a sub matrix of A, if A_{11} is nonsingular and B_{ij} is a sub matrix of B, if B_{11} is nonsingular then considering (4.1) as simultaneous equations we obtain

$A_{11}x^{(1)} + A_{12}x^{(2)} = b_1$…(5.1)

$A_{21}x^{(1)} + A_{22}x^{(2)} = b_2$…(5.2)

Eliminating $x^{(1)}$ from (5.1) and substituting in (5.2) we get

$x^{(1)} = A_{11}^{-1}(b_1 - A_{12}x^{(2)})$…(6.1)

$(A_{22} - A_{21}A_{11}^{-1}A_{12})x^{(2)} = [b_2 - (A_{21}A_{11}^{-1})b_1]$ … (6.2)

Similarly

$y^{(1)} = B_{11}^{-1}(g_1 - B_{12}y^{(2)})$…(7.1)

$(B_{22} - B_{21}B_{11}^{-1}B_{12})y^{(2)} = [g_2 - (B_{21}B_{11}^{-1})g_1]$ … (7.2)

Substituting x in (2.3) we get .

IV. NUMERICAL EXAMPLES

In this section we apply the algorithm for solving FFLS of fuzzy triangular numbers.

EXAMPLE :1

Consider the FFLS

$$\begin{bmatrix} (1,1,1) & (8,5,9) \\ (6,4,4) & (2,2,2) \end{bmatrix} \begin{bmatrix} (x_1, y_1, z_1) \\ (x_2, y_2, z_2) \end{bmatrix} = \begin{bmatrix} 23,33,62 \\ 46,24,70 \end{bmatrix}$$

Solution:

$$A = \begin{bmatrix} 1 & 8 \\ 6 & 2 \end{bmatrix}, B = \begin{bmatrix} 1 & 5 \\ 4 & 2 \end{bmatrix}, M = \begin{bmatrix} 1 & 9 \\ 4 & 2 \end{bmatrix}$$

$$b = \begin{bmatrix} 23 \\ 46 \end{bmatrix}, g = \begin{bmatrix} 33 \\ 24 \end{bmatrix}, h = \begin{bmatrix} 62 \\ 70 \end{bmatrix}$$

Using $(\mathbf{4.1})$

We get $A = \begin{bmatrix} 1 & 8 \\ 6 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 23 \\ 46 \end{bmatrix}$

$A_{11} = 1, A_{12} = 8, A_{21} = 6, A_{22} = 2, b_1 = 23, b_2 = 46$

We get $x_1 + 8x_2 = 23$, $6x_1 + 2x_2 = 46$

On solving $X = \begin{bmatrix} 7 \\ 2 \end{bmatrix}$

Using (4.2) we get $= \begin{bmatrix} 1 & 5 \\ 4 & 2 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 33 \\ 24 \end{bmatrix}$

$B_{11} = 1, B_{12} = 5, B_{21} = 4, B_{22} = 2, g_1 = 33, g_2 = 24$

We get

$y_1 + 5y_2 = 33$, $4y_1 + 2y_2 = 24$

On solving $Y = \begin{bmatrix} 3 \\ 6 \end{bmatrix}$

Using (3.3)

We get $\begin{bmatrix} 1 & 8 \\ 6 & 2 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} + \begin{bmatrix} 1 & 9 \\ 4 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 62 \\ 70 \end{bmatrix}$

Substituting x_1, x_2 values we get $z_1 + 8z_2 = 37,\ 6z_1 + 2z_2 = 38$ on solving $z = \begin{bmatrix}5\\4\end{bmatrix}$

Hence the solution of the fully fuzzy linear system is $\tilde{x} = \begin{bmatrix}(7,3,5)\\(2,6,4)\end{bmatrix}$

EXAMPLE: 2

Consider the FFLS

$$\begin{bmatrix}(1,4,4) & (2,6,6) & (3,6,1)\\(2,6,6,) & (8,18,4) & (8,12,4)\\(3,6,1) & (8,12,4) & (14,9,4)\end{bmatrix}\begin{bmatrix}(x_{1,}y_{1,}z_1)\\(x_{2,}y_{2,}z_2)\\(x_{3,}y_{3,}z_3)\end{bmatrix} = \begin{bmatrix}(10,36,172)\\(20,42,216)\\(30,40,56)\end{bmatrix}$$

Solution:

Similar procedure to follow, that justifies the proposed illustration above.

V. CONCLUSION

In this paper solution of fully fuzzy linear system $\tilde{A} \otimes \tilde{x} = \tilde{b}$ is obtained by partitioning the coefficient matrix into sub matrices using Schur complement by a new methodology in the form of triangular fuzzy number matrices. This method is very efficient for the system $\tilde{A} \otimes \tilde{x} = \tilde{b}$. when very large. This method is easy to implement in parallel computing.

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