Solving a Decision Making Problem Using Weighted Fuzzy Soft Matrix

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ABSTRACT
The purpose of this paper is to use soft set theory in decision making in banking system. A new efficient solution procedure has been developed to solve fuzzy soft set based real life decision making problems involving multiple decision makers. In this paper new technique of applying threshold for selecting an optimal set is suggested when the user deals with huge amount of data and optimal subset of the data is to be selected.

Key words: *Fuzzy Sets, Fuzzy Soft Set Applications, Soft Sets, Soft Set Matrices.*

1. INTRODUCTION
Decision making is vital in today's fast moving world. It is significant for all categories of problems dealing with the problems in Engineering, Medical, Social Sciences, and Management etc. It involves the selection from two or more alternative courses of action. The decision maker [1] is presented with alternative courses of action who then selects the best one which meets the objectives of the problem satisfactorily based on logical and qualitative analysis. The inherent problem of decision making is related to vagueness and uncertainty aspects. Classical mathematics is not very effective in dealing with such type of problems. Some theories like probability theory, fuzzy set theory [2], intuitionistic fuzzy set [3], rough set theory [4], have tried to resolve this problem. There are some popular mathematical tools which deal with uncertainties; two of them are fuzzy set theory, which was developed by Zadeh (1965), and soft set theory, which was introduced by Molodtsov (1999), that are related to this work. Though all these techniques do not consider parameterization of the tools hence could not be applied truly in handling problems of uncertainties. The soft set concepts is devoid of all these problems and possess rich potential of solving certain decision making problem [5] involving recruitment problem, investment problem or selecting customer for subscription. Currently, work on the soft set theory is rapidly progressing. Maji et al (2003) defined operations on soft sets and made a detailed theoretical study on that. By using these definitions, the applications of soft set theory have been studied increasingly [6] [7] [8] [9] [10].

2. PRELIMINARIES
Here we will describe the preliminary definitions, and results which will be required later in this paper.

2.1 Fuzzy Set
Fuzzy sets are the ones in which elements have degrees of membership. This was introduced as an extension of the classical notion of set by Lotfi A. Zadeh [2] and Dieter Klaua [4] in 1965. Fuzzy relations, which are used now in different areas, such as linguistics (De Cock, et al, 2000), decision-making (Kuzmin, 1982) and clustering (Bezdek, 1978), are special cases of L-relations when L is the unit interval [0, 1].In classical set theory, the membership of elements in a set is in terms of binary terms based on a condition that an element will either belongs to or will not belong to the set. In contrast, fuzzy set theory allows the gradual assessment of the membership of elements in a set; this has been described with the aid of a membership function valued in the real unit interval [0, 1]. Fuzzy sets are the ones that generalize the classical sets, since the characteristic function of classical sets are special cases of the membership functions of fuzzy sets as the classical set only take values 0 or 1. In fuzzy set theory, classical bivalent sets are generally called crisp sets. The fuzzy set theory is widely used in domains where information is incomplete or imprecise, like in an area of bioinformatics, medicine, and banking. In this subsection, the basic definitions of fuzzy set theory (Zadeh, 1965) are described which will be useful for subsequent discussions. The detailed description related to this theory is also available in earlier studies (Dubois & Prade 1980, Klir & Folger 1988, Zimmermann 1991).

Definition:
Let X be a space of points, with a generic element of X denoted by x. Thus X = {x}.A fuzzy set A in X is characterized by a membership function fA(x) which associates with each point in X a real number in the interval [0,1], with the values of fA(x) at x representing the "grade of membership"

of x in A. Thus, the nearer the value of fA(x) to unity, the higher the grade of membership of x in A.

2.2 Soft Set

Soft set theory is a generalization of fuzzy set theory which was proposed by Molodtsov in 1999 to deal with uncertainty in a non-parametric manner. Mathematically, a soft set is defined as if X is a universal set and set of parameters E is a pair (f, A) where f is a function and A is a set such that f(e) is a subset of the universe X, where e is element of the set A. For each e the set f (e) is called the value set of e in (f, A).

Definition: Let U be an initial universe, P (U) be the power set of U, E be the set of all parameters and A $\subseteq$E .Parameters are the attributes, properties or characteristics of objects. Then, a soft set FA over U is a set defined by a function f_A representing a mapping $f_A : E \to P(U)$ such that $f_A(x) = \varnothing$ if $x \notin A$.

Example 2.2.1: Suppose that $U = \{c_1, c_2, c_3, c_4\}$ is a set of four sarees and $E = \{e_1, e_2, e_3\}$ is a set of parameters, which stand for white, red and blue respectively. If $A = \{e_1, e_2\} \subseteq E$ and let $F_A = \{c_1, c_2, c_3\}$, then $(F_A, E) = \{\{e_1, (c_1, c_2, c_3)\}, \{e_2, (c_1, c_2, c_3)\}\}$ over U which describe the "colour of sarees". This soft set representation is shown in the Table 1 below.

Table 1: Soft Set

U	White (e_1)	Red (e_2)	Blue(e_3)
c_1	1	1	0
c_2	1	1	0
c_3	1	1	0
c_4	0	0	0

2.3 Fuzzy Soft Set

In this section we briefly explain the concept of fuzzy soft which is certain extensions of the crisp soft set. The fuzziness or vagueness deals with uncertainty inherent in the Decision Making Problems of the real world. The definition of fuzzy soft set is followed by an example.

Definition: Let P (U) denotes the set of all fuzzy sets of U. Let $Ai \subseteq E$.A pair (Fi, Ai) is called a fuzzy -soft-set over U, where Fi is a mapping given by $Fi: Ai \to P\ (U)$.

U	White (e_1)	Red (e_2)	Blue (e_3)
c_1	0.9	0.8	0
c_2	0.3	0.9	0
c_3	0.8	0.4	0
c_4	0.9	0.3	0

Example 2.3.1: In the example of Soft Set considered above, it will not be possible to express it with only the two numbers 0 and 1.In that case we can characterize it by a membership function instead of the crisp number 0 and 1,that associates with each element a real number in the interval [0,1].The fuzzy soft set can then be described as where A={ e_1,e_2} (F, A)={F(e_1) = {(c_1,0.9), (c_2,0.3), (c_3,0.8), (c_4,0.9)}, F(e_2) = {(c_1,0.8), (c_2,0.9), (c_3,0.4), (c_4,0.3)}}

This fuzzy soft set representation is shown in the Table 2 below.

Table 2: Fuzzy Soft Set

2.4 Fuzzy Soft Matrix

In this section we give the definition of fuzzy parameterized soft set we give examples for these concepts.

Definition: Let U be an initial universe, P (U) be the power set of U, E be the set of all parameters and X be a fuzzy set over E with the membership function $\mu_X : E \to [0,\ 1]$.

Then, an fps-set F_X over U is a set defined by a function f_X representing a mapping

$f_X : E \to P(U)X$ such that $f_X(x) = \varnothing$ if $\mu_X(x) = 0$. Here, f_X is called approximate function of the fps-set F_X, and the value $f_X(x)$ is a set called x-element of the fps-set for all $x \in E$. Thus, an fps-set F_X over U can be represented by the set of ordered pairs

$$F_X = \{(\mu_X(x)/x, f_X(x)): x \in E, f_X(x) \in P(U), \mu_X(x) \in [0,1]\}$$

Example2.4.1: Let $U = \{c_1, c_2, c_3 \ldots c_m\}$ be the Universal set and E be the set of parameters given by $E = \{e_1, e_2, e_3 \ldots e_n\}$.Let $A \subseteq E$ and (F, A) be a fuzzy soft set in the fuzzy soft class (U, E).Then fuzzy soft set (F, A) in a matrix form as $A_{mXn} = [a_{ij}]_{mXn}$

$A = [a_{ij}]$ $i = 1, 2 \ldots m, j = 1, 2, 3 \ldots n$ Where $a_{ij} = \mu_j(c_i)$ if $e_j \in A$ OR $a_{ij} = 0$ if $e_j \notin A$ $\mu_j(c_i)$ represents the membership of c_i in the fuzzy set $F(e_j)$.

From the example 2.4.1, the matrix is represented as

$$\begin{pmatrix} 0.9 & 0.8 & 0 \\ 0.3 & 0.9 & 0 \\ 0.8 & 0.4 & 0 \\ 0.9 & 0.3 & 0 \end{pmatrix}$$

3. FUZZY WEIGHTED SOFT MATRIX IN DECISION MAKING

In this section, we have put forward a weighted fuzzy soft matrix decision making method by using fuzzy soft sets and then we have discussed and applied it to decision-making problem. The idea of weighted fuzzy parameterized soft matrix set provides a mathematical framework for modelling and analyzing the decision-making problems in which all the parameters may not be of equal importance. These differences between the importances of parameters are characterized by the weight function in a weighted fuzzy parameterized soft matrix set.

Algorithm

Input: Fuzzy soft sets with r objects, each of which has s parameters.

Output: An optimal set

1. Get the Universal set having r objects

2. Choose the set of parameters

3. Consider the weights to be applied for each set of parameters based on the expert's decision and relevance of the attribute (parameter)

4. Compute the arithmetic mean of membership and non-membership value of fuzzy soft matrix as A_{AM}

5. Assign Weights to each set of parameters based on the importance and thus compute the weighted arithmetic mean.

6. Choose the object with highest membership value.

7. In case of tie i.e. when more than one object with same highest membership value , choose the object with highest membership value as well as lowest non-membership value.

8. In case of applications which involve decision making of selecting large optimal number of persons, some threshold value could be set. The objects above that threshold value could be selected and the ones below the threshold could be rejected.

9. Thus the optimum decision set could be obtained.

4. PROBLEM

The development of a weighted fuzzy soft set based selection strategy is described for selection of customers whom the bank should target for deposit subscription [11]. Fuzzy soft set theory aims to model imprecise, vague and fuzzy information. Computers cannot adequately handle such problems, because machine intelligence still employs sequential (Boolean) logic. The business goal is to find an optimal subset that can explain success of a contact, i.e. if the client subscribes for the loan or can be specified as a good customer for further business deals. Such optimal set can increase campaign efficiency by identifying the main characteristics that affect success, helping in a better management of the available resources (e.g. human effort, phone calls, time) and selection of a high quality and affordable set of potential buying. Consider the huge set of objects which in the current application are the customers whom we are selecting and segregating from the universal set. Let these be represented by (c1, c2, c3, c4……cm).Then we have the following six inputs are taken for consideration i.e. Middle aged, average yearly balance, existence of loan, duration of contact, in-come, marital status". This data reflects the degree of vagueness in the information collected by the banks from various sources and furnished by the customer. Based on the consultation with the banking expert, the author has selected only four parameters (e1=middle aged; e2=average yearly balance; e3=existence of loan; e4=income) for further processing.

Let E = (e1,e2,e3,e4,e5,e6) be the set of parameters and P={e1,e2,e3,e4} $\subseteq$ E

(F, P) = {{F (e1) = {(c1, 0.3), (c2, 0.4), (c3, 0.2), (c4, 0.4)},
{F (e2) = {(c1, 0.2), (c2, 0.4), (c3, 0.2), (c4, 0.5)},
{F (e3) = {(c1, 0.7), (c2, 0.2), (c3, 0.6), (c4, 0.4)},
{F (e4) = {(c1, 0.5), (c2, 0.6), (c3, 0.4), (c4, 0.2)}}

Then

$$A = \begin{pmatrix} 0.3 & 0.2 & 0.7 & 0.5 \\ 0.4 & 0.4 & 0.2 & 0.6 \\ 0.2 & 0.2 & 0.6 & 0.4 \\ 0.4 & 0.5 & 0.4 & 0.2 \end{pmatrix}$$

$$A_{AM} = \begin{pmatrix} 0.425 \\ 0.4 \\ 0.35 \\ 0.375 \end{pmatrix}$$

If the buyer X provides weights as 0.3, 0.1, 0.2, 0.4 on "middle aged", "average yearly balance", "existence of loan", "income" respectively, then

$$A_{WAM} = \begin{pmatrix} 0.1275 \\ 0.04 \\ 0.07 \\ 0.15 \end{pmatrix}$$

This calculation is shown only for four customers. This can be extended to n customers. Now we need to select the optimal customers who could be selected for promotional marketing campaign. Based on the consultation with the banking expert the user can select a threshold value. In the present scenario if the threshold values are set as 0.12 then customer c1 and c4 could be selected and customer's c2 and c3 would be rejected as it lies below the threshold.

5. CONCLUSION

The use of a weighted fuzzy soft matrix set based system for targeting specific customer is taken into consideration. The system proposed through this work is evaluated on hypothetical data. In this paper, we have introduced the concept of setting threshold when large numbers of customers have to be selected and also used the earlier concept of soft set by assigning weights based on the relevance of attributes. In this, weighted arithmetic mean has been used to derive the decision factors on the fuzzy soft matrix set. This method can be further applied on other decision making problem having uncertain parameters.

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