# APPLICATION OF MODIFIED EULER'S METHOD IN OBTAINING NUMERICAL SOLUTION OF SWING EQUATION

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## ABSTRACT

The stability of a system refers to the ability of a system to return back to its steady state when subjected to a disturbance. As mentioned before, power is generated by synchronous generators that operate in synchronism with the rest of the system. A generator is synchronized with a bus when both of them have same frequency, voltage and phase sequence. We can thus define the power system stability as the ability of the power system to return to steady state without losing synchronism. Usually power system stability is categorized into steady state, transient and dynamic stability. Transient stability analysis is the study of the system stability when subjected to server faults like three-phase to ground short circuit or major generator outage etc.

The large-disturbance (here after will be called as transient stability) of a Single Machine Infinite Bus (SMIB) system can be analyzed through a method called as "equal area criterion". Equal area criterion can be obtained from the swing equation. The equal area criterion gives an analytic way of assessing the stability of the system. The transient stability of the system can also be found out by numerically integrating the swing equation. Since, swing equation is nonlinear we cannot solve for the solution of swing equation through analytical methods. In order to solve the swing equation numerical methods have to be used. In this article a look at a well known numerical method called as Euler's method is given for the numerical solution of Swing equation.

## INTRODUCTION

*Power system stability* is understood as the ability to regain an equilibrium state after being subjected to a physical disturbance. Synchronous stability can be divided into two main categories depending upon the magnitude of disturbance. Steady State stability refers to the ability of the power system to regain synchronism after small and slow disturbance, such as gradual power changes. The transient stability is the ability of the system to regain synchronism after a large disturbance. Transient stability studies are needed to ensure that the system can withstand the transient conditions following a major disturbance.

Steady State stability of the power system is analyzed by the swing equation of a synchronous machine. It represents the swings in the rotor angle$\delta$during disturbances. The change in the rotor angle $\delta$ results in change in real power, which ultimately affects the frequency. Hence swing equation forms the basis for modelling of load frequency control (LFC) loop of the power system. Swing equation is a rotational inertia equation describing the effect of unbalance between the electromagnetic torque and the mechanical torque of the individual machines.

## SWING EQUATION

The behaviour of a synchronous machine during transients is described by the swing equation. Let θ be the angular position of the rotor at any instant t.However, θ is continuously changing with time. It is convenient to measure θ with respect to reference axis that is rotating at synchronous speed. If $\delta$ is the angular displacement of the rotor in electrical degrees from the synchronously rotating reference axis and $\omega_s$ the synchronous speed in electrical radians ,then θ can be expressed as the sum of time varying angle $\omega_s$ t on the rotating reference axis ,plus the torque angle $\delta$ of the rotor with respect to the rotating reference axis.

In other words:

$\theta = \omega_s t + \delta$ radians. .............. 1

Differentiating equation- 1 with respect to t we get

$$\frac{d\theta}{dt} = \omega_s + \frac{d\delta}{dt} \quad ............... \quad 2$$

Differentiation of equation- 2 gives

$$\frac{d^2\theta}{dt^2} = \frac{d^2\delta}{dt^2}$$

Angular acceleration of rotor

$$\alpha = \frac{d^2\theta}{dt^2} = \frac{d^2\delta}{dt^2} \text{elec. rad/s}^2$$

If damping is neglected the accelerating torque, in a synchronous generator is equal to the difference of input mechanical or shaft torque and the output electromagnetic torque .

That is, $T_a = T_s - T_e$ ............. 3

Let $\omega$ = synchronous speed of the rotor,
J = moment of inertia of the rotor
M = angular momentum of the rotor
$P_s$ = mechanical power input
$P_e$ = electrical power output.
$P_a$ = accelerating power.

Now $M = J\omega$

Multiply both the sides of equation- 3 by $\omega$ we get,

$$\omega T_a = \omega T_s - \omega T_e$$

$$P_a = P_s - P_e$$

But , $J\frac{d^2\theta}{dt^2} = T_a$ ;

$$J\frac{d^2\delta}{dt^2} = T_a$$

$$\omega J\frac{d^2\delta}{dt^2} = \omega T_a$$

$$M\frac{d^2\delta}{dt^2} = P_a = P_s - P_e \quad ............4$$

Equation-4 gives the relation between the accelerating power and angular acceleration.
It is called the swing equation. It is a non-linear differential equation of the second order. With this differential equation we can discuss the stability in a quantitative way,because it describes swings in the power angle delta during transients.

**Numerical Integration of Swing Equation**

The Equal Area Criterion gives an analytic way of assessing the stability of the system. The transient stability of the system can also be found out by numerically integrating the swing equation. Since, swing equation is nonlinear we cannot solve for the solution of swing equation through analytical methods. In order to solve the swing equation numerical methods have to be used.

Let us look at a well known numerical method called as Euler's method.

**Euler's method**

Let a non linear differential equation of the form given in (2.58) exist, where $f(x)$ is a nonlinear function of $x$

$$\frac{dx}{dt} = f(x)$$

We can find the solution of above equation through Euler's method. The idea is to integrate the equation between the time instants ($t_0$,$t_f$) with an initial value of $x = x^0$ at $t = t_0$ with a small time step $\Delta t$ .

This can be expressed as:

$$x^1 = x^0 + \left( \left. \frac{dx}{dt} \right|_{x=x^0} \right) \Delta t$$

Here, $x^1$ is the variable value at time instant $t = t_0 + \Delta t$. This process has to be repeated iteratively until the final time $t_f$ is reached. Above equation is nothing but Taylor series expansion of the variable $x$ around the operating point ($t_0$, $x^0$) with higher order terms discarded. Since, the higher terms are discarded the above equation may introduce error. Hence, to reduce the error modified Euler's method can be used.

Modified Euler's method has two steps: predictor step and corrector step. These steps in generalized form are as follows:

$$x_p^{k+1} = x^k + \left( \left. \frac{dx}{dt} \right|_{x=x^k} \right) \Delta t$$

$$x_c^{k+1} = x^k + \frac{1}{2} \left( \left. \frac{dx}{dt} \right|_{x=x_p^{k+1}} + \left. \frac{dx}{dt} \right|_{x=x^k} \right) \Delta t$$

Where $x_p^1$ is the predicted and $x_c^1$ is the corrected value of the variable $x$ at the time instant $t = t_0 + \Delta t$. The time step $\Delta t$ has to be carefully chosen,the smaller the values the better the accuracy. Ideally $\Delta t$ should approach zero but when the number of steps required to integrate above equation between the limits $(t_0, t_f)$ will become infinite. Hence there is a trade off between the accuracy and the number of steps required. Now modified Euler's method can be applied to integrate swing equation between the time limits $(t_0, t_f)$. The second order differential equations of the single machine infinite bus system are given below:

$$\frac{d\delta}{dt} = \omega_{me} - \omega_s = \Delta\omega_{me}$$

$$\frac{d\Delta\omega_{me}}{dt} = \frac{\pi f_s}{H}\left(P_m - P_{\max}\sin\delta\right)$$

Here, $\Delta\omega_{me} = \omega_{me} - \omega_s$ that is the change in the rotor speed from the synchronous speed. Now the predictor step of Euler's method when applied to eq will give:

$$\delta_p^{\,k+1} = \delta^k + \left.\frac{d\delta}{dt}\right|_{\substack{\delta=\delta^k \\ \Delta\omega_{me}=\Delta\omega_{me}^k}} \Delta t$$

$$\Delta\omega_{me\,p}^{\;k+1} = \Delta\omega_{me}^k + \left.\frac{d\Delta\omega_{me}}{dt}\right|_{\substack{\delta=\delta^k \\ \Delta\omega_{me}=\Delta\omega_{me}^k}} \Delta t$$

The corrector step is given as:

$$\delta_c^{\,k+1} = \delta^k + \frac{1}{2}\left( \left.\frac{d\delta}{dt}\right|_{\substack{\delta=\delta_p^{k+1} \\ \Delta\omega_{me}=\Delta\omega_{mep}^{k+1}}} + \left.\frac{d\delta}{dt}\right|_{\substack{\delta=\delta^k \\ \Delta\omega_{me}=\Delta\omega_{me}^k}} \right)\Delta t$$

$$\Delta\omega_{me\,c}^{\;k+1} = \Delta\omega_{me}^k + \frac{1}{2}\left( \left.\frac{d\Delta\omega}{dt}\right|_{\substack{\delta=\delta_p^{k+1} \\ \Delta\omega_{me}=\Delta\omega_{mep}^{k+1}}} + \left.\frac{d\Delta\omega}{dt}\right|_{\substack{\delta=\delta^k \\ \Delta\omega_{me}=\Delta\omega_{me}^k}} \right)\Delta t$$

By numerically integrating the swing equation for a specified period like for 5 seconds,if we observe that the rotor angle and the rotor speed are settling to the steady state values when subjected to a disturbace then we can say that the system is stable. But if we observe from numerical integration of swing equation that the rotor angle and speed are either continuously increasing or decreasing as time tends towards infinity then that means that the system has become unstable.

**Example**: A 50 Hz ,synchronous generator having inertia constant H= 5.2 MJ/VA and xd'= 0.3 pu is connected to an infinite bus through a double circuit line .The reactance of the connecting HT Transformer is 0.2 pu and the reactance of each line is 0.4 pu.$|E_g|$= 1.2 pu and $|V|$= 1.0 pu. To obtain the swing curve using modified Euler's method for a three phase fault occurs at the middle of one of the transmission lines and is cleared by isolating the faulted line.

To solve the swing equation by Modified Euler‟s method, it is written as two first order differential equations:

$$\frac{d\delta}{dt} = \omega$$

$$\frac{d\omega}{dt} = \frac{P_a}{M} = \frac{P_m - P_{\max}\sin\delta}{M}$$

Starting from an initial value at the beginning of any time step, and choosing a step size, the equations to be solved in modified Euler's are as follows:

$$\left.\frac{d\delta}{dt}\right|_0 = D_1 = \omega_0$$

$$\left.\frac{d\omega}{dt}\right|_0 = D_2 = \frac{P_m - P_{\max}\sin\delta_0}{M}$$

$$\delta^P = \delta_0 + D_1\,\Delta t$$

$$\omega^P = \omega_0 + D_2\,\Delta t$$

$$\left.\frac{d\delta}{dt}\right|_P = D_{1P} = \omega^P$$

$$\left.\frac{d\omega}{dt}\right|_P = D_{2P} = \frac{P_m - P_{\max}\sin\delta^P}{M}$$

$$\delta_1 = \delta_0 + \left(\frac{D_1 + D_{1P}}{2}\right)\Delta t$$

$$\omega_1 = \omega_0 + \left(\frac{D_2 + D_{2P}}{2}\right)\Delta t$$

$\delta_1$ and $\omega_1$ are used as initial values for the successive time step..
Applying the above equations for solving we have,

Table 1: Calculation of derivatives in Modified Euler's Method

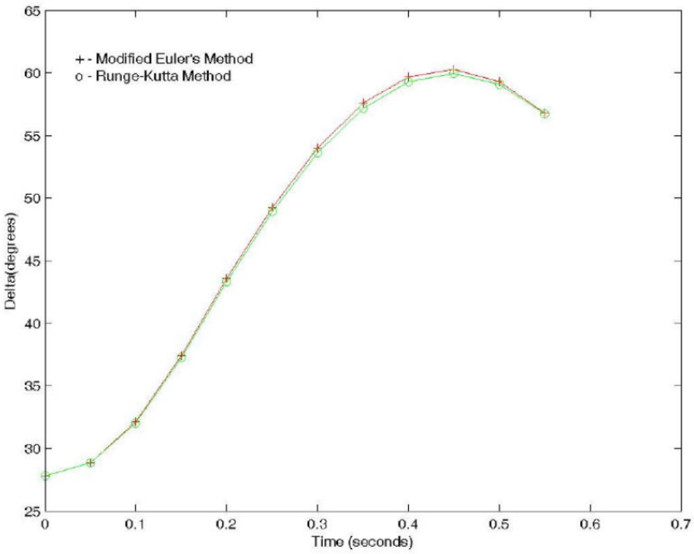
| t | $D_1$ | $D_2$ | $\delta^P$ | $\omega^P$ | $D_{1P}$ | $D_{2P}$ |
|---|---|---|---|---|---|---|
| $0^+$ | 0.0 | 15.296 | 0.485 | 0.765 | 0.765 | 15.296 |
| 0.05 | 0.765 | 14.977 | 0.542 | 1.514 | 1.514 | 14.350 |
| 0.10 | 1.498 | 14.043 | 0.636 | 2.200 | 2.200 | 12.860 |
| 0.15 | 2.17 | - 0.299 | 0.761 | 2.155 | 2.155 | - 3.600 |
| 0.20 | 2.072 | - 3.604 | 0.865 | 1.892 | 1.892 | - 6.482 |
| 0.25 | 1.820 | - 6.350 | 0.951 | 1.502 | 1.502 | - 8.612 |
| 0.30 | 1.446 | - 8.424 | 1.015 | 1.025 | 1.025 | - 10.041 |
| 0.35 | 0.984 | - 9.827 | 1.054 | 0.493 | 0.493 | - 10.843 |
| 0.40 | 0.467 | - 10.602 | 1.065 | - 0.063 | - 0.063 | - 11.060 |
| 0.45 | - 0.074 | - 10.803 | 1.048 | - 0.614 | - 0.614 | - 10.720 |
| 0.50 | - 0.612 | - 10.46 | 1.004 | - 1.135 | - 1.135 | - 9.800 |

Table 2: Calculation of derivatives in Modified Euler's Method

| T | $P_{max}$ (pu) | $\delta_o$ rad | $\omega_o$ rad / sec | $\delta_1$ rad | $\omega_1$ rad / sec | $\delta_1$ deg |
|---|---|---|---|---|---|---|
| $0^-$ | 1.714 | 0.485 | 0.0 | – | – | - |
| $0^+$ | 0.630 | 0.485 | 0.0 | 0.504 | 0.765 | 28.87 |
| 0.05 | 0.630 | 0.504 | 0.765 | 0.561 | 1.498 | 32.14 |
| 0.10 | 0.630 | 0.561 | 1.498 | 0.653 | 2.170 | 37.41 |
| 0.15 | 1.333 | 0.653 | 2.170 | 0.761 | 2.072 | 43.60 |
| 0.20 | 1.333 | 0.761 | 2.072 | 0.860 | 1.820 | 49.27 |
| 0.25 | 1.333 | 0.860 | 1.820 | 0.943 | 1.446 | 54.03 |
| 0.30 | 1.333 | 0.943 | 1.446 | 1.005 | 0.984 | 57.58 |
| 0.35 | 1.333 | 1.005 | 0.984 | 1.042 | 0.467 | 59.70 |
| 0.40 | 1.333 | 1.042 | 0.467 | 1.052 | - 0.074 | 60.27 |
| 0.45 | 1.333 | 1.052 | - 0.074 | 1.035 | - 0.612 | 59.30 |
| 0.50 | 1.333 | 1.035 | - 0.612 | 0.991 | - 1.118 | 56.78 |

## CONCLUSIONS

The swing curves obtained from modified Euler's method and Runge-Kutta method are shown in figure below. It can be seen that the two methods yield very close results.




Euler's method is one of the easiest methods to program for solution of differential using a digital computer. It uses the Taylor's series expansion, discarding all second –order and higher order terms. Modified Euler's algorithm uses the derivatives at the beginning of a time step, to predict the values of the dependent variables at the end of the step.Using the predicted values, the derivatives at the end of the interval are computed. The average of the two derivatives is used in updating the variables.

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