A DISTINCT METHOD TO FIND THE CRITICAL PATH AND TOTAL FLOAT UNDER FUZZY ENVIRONMENT

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ABSTRACT
In this paper, A new method has been proposed to find the critical path and the total float for the project scheduling problems with the aid of Interval valued fuzzy numbers (IVFNS). A new distance function with ranking function has also been utilized to find the critical path. A relevant numerical illustration is also included to explain the above said notion.

Keywords *- Ranking function, Interval valued fuzzy numbers (IVFNS), Operations on IVFNS, Project scheduling, Distance function.*

I. INTRODUCTION

Fuzzy set theory has been proposed to handle non crisp parameters by generalizing the notion of membership in a set. Essentially in a fuzzy set each element is associated with a point value selected from the unit interval [0,1] which is an arbitrary grade of truth referred to as the grade of membership in the set. The main object is to find the best solution possible with imprecise, vague, uncertain or incomplete information.

A project network is defined as a set of activities that must be performed according to precedence constraints stating which activities must start after the completion of specified other activities[9]. A path through a project network is one of the routes from the starting node to the ending node. According to the critical path the length of a path is the sum of the durations of the activities on the path.

Nasution [10] described the method to find the critical path method. Ravishankar and sireesha [4] extended a graphical approach to find the critical path in a project network. Stephen Dinagar and Abirami [5,6] discussed a fuzzy critical path analysis in project network on L-R type interval valued fuzzy numbers. Zareeri [2] gave a new approach for solving fuzzy critical path problem using analysis of events. In [1], Thangaraj Beaula and V.Vijaya studied a new method for finding the critical path in a fuzzy project network.

In this paper, we introduced a new method to finding the critical path by using the minimum path length dealing with interval valued fuzzy networks. The approach is illustrated by suitable examples. The paper is organized as follows. Firstly in section 2, we presented the definitions of interval valued fuzzy numbers and their arithmetic operations on IVFNS. In section 3, a new algorithm to find the critical path is introduced. In section 4, a relevant numerical example is given. In section 5, conclusion is also given.

II. PRELIMINARIES

In this section, some basic definitions and arithmetic operations are reviewed.

Definition. 2.1

A fuzzy set $\tilde{A}$ in a universe of discourse X is defined as the following set of pairs $\tilde{A} \equiv \{(x, \mu_{\tilde{A}}(x)) : x \epsilon X\}$. Here $\mu_{\tilde{A}}$:x→ $[0,1]$ is a mapping called the membership value of xϵX in a fuzzy set $\tilde{A}$.

Definition.2.2

The α_level set (or interval of confidence at level α (or α_cut) of the fuzzy set $\tilde{A}$ of X is a crisp set A_α that contains all the elements of X that have membership values in A greater than or equal to α.

i.e., $A_\alpha = \{x : \mu_{\tilde{A}}(x) \geq \alpha, x \in X, \alpha \in [0,1]\}$

Definition.2.3 [3]

A fuzzy set $\tilde{A}$, defined on the set of real numbers R is said to be a fuzzy number if its membership function has the following characteristics.

(i) $\tilde{A}$ is convex, i.e.,$\tilde{A}$ $(\lambda X_1 + (1-\lambda) X_2)$ = Minimum {$\tilde{A}$ (X_1), $\tilde{a}$ (X_2)}, for all $X_1,X_2 \in$ R and $\lambda \in [0,1]$
(ii) $\tilde{A}$ is normal i.e., there exists an $X_0 \in$ R such that $\tilde{A}$ $(X_0) = 1$
(iii) $\tilde{A}$ is piecewise continuous.

Definition.2.4

An Interval valued fuzzy number $\tilde{\mathrm{A}}$ on R is given by
$\tilde{\mathrm{A}}$= { x,($\mu_{\tilde{A}}^L$(x),$\mu_{\tilde{A}}^U$(x)), x∈R } and $\mu_{\tilde{A}}^L(x) \leq \mu_{\tilde{A}}^U(x)$ for all x∈R.
And it is denoted by $\tilde{\mathrm{A}} = [\tilde{\mathrm{A}}^L, \tilde{\mathrm{A}}^U]$,
Where $\tilde{\mathrm{A}}^L$ = (a_1^L, a_2^L, a_3^L, a_4^L) and $\tilde{\mathrm{A}}^U$= (a_1^U, a_2^U, a_3^U, a_4^U) are the trapezoidal fuzzy numbers.
It is also noted that $a_1^U \leq a_1^L$, $a_2^U \leq a_2^L$, $a_3^L \leq a_3^U$, $a_4^L \leq a_4^U$

Pictorial Representation: 2.5

Let $\hat{A}$= [(2,4,5,7), (1,3,6,8)]

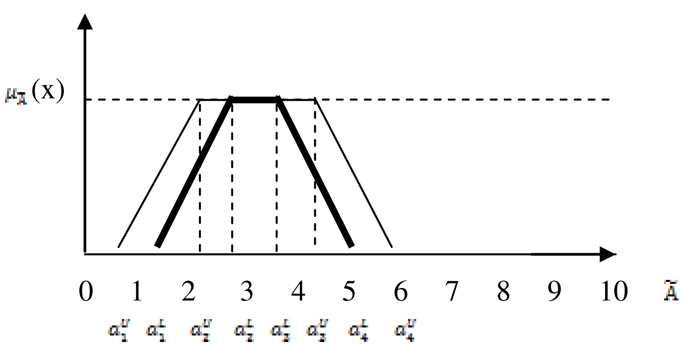

Fig 2.1 – IVFN $\tilde{\mathrm{A}}$

Definition .2.6. [11]

An efficient for comparing the fuzzy number is by the use of ranking function defined R:F(R)→R,where F(R) is a set of fuzzy numbers defined on a set of real numbers, which maps each fuzzy number into a real number where a natural order exists.

For $\tilde{\mathrm{A}} = (a_1^L, a_1^U) \epsilon F(R)$,then the ranking function $\mathcal{R}$:F(R)→R is defined as:

$\mathcal{R}(\tilde{\mathrm{A}}) = (a_1^L+a_2^L+a_3^L+a_4^L+a_1^U+a_2^U+a_3^U+a_4^U)/8$

Definition.2.7

A fuzzy number $\tilde{\mathrm{A}}$ is said to be trapezoidal fuzzy number if its membership function

$\mu_{\tilde{A}}$:x→ [0,1] has the following characteristic function:

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{(x-a_1)}{(a_2-a_1)}, & a_1 \leq x \leq a_2 \\ 1, & a_2 \leq x \leq a_3 \\ \frac{(x-a_4)}{(a_3-a_4)}, & a_3 \leq x \leq a_4 \\ 0, & otherwise \end{cases}$$

Definition.2.8 [7]

A fuzzy number $\tilde{\mathrm{A}}$ is said to be an Interval valued fuzzy number if its membership function $\mu_{\tilde{A}}$:x→ [0,1] has the following characteristic function:

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-a_1^L}{a_2^L-a_1^L}, & a_1^L \leq x \leq a_2^L \\ \frac{a_4^L-x}{a_4^L-a_3^L}, & a_3^L \leq x \leq a_4^L \\ 1, & a_2^U \leq x \leq a_3^U \text{ and } a_2^L \leq x \leq a_3^L \\ \frac{x-a_1^U}{a_2^U-a_1^U}, & a_1^U \leq x \leq a_2^U \\ \frac{a_4^U-x}{a_4^U-a_3^U}, & a_3^U \leq x \leq a_4^U \\ 0, & otherwise \end{cases}$$

Arithmetic Operations on IVFNS.2.9 [6]

In this section, arithmetic operations between two Interval valued fuzzy numbers, defined on universal set of real numbers R.

Let $\tilde{\mathrm{A}}$ = [(a_1^L,a_2^L,a_3^L,a_4^L), (a_1^U,a_2^U,a_3^U,a_4^U)] and
$\tilde{\mathrm{B}}$ = [(b_1^L,b_2^L,b_3^L,b_4^L), (b_1^U,b_2^U,b_3^U,b_4^U)] then we define

(i) Addition:
$\tilde{\mathrm{A}} \oplus \tilde{\mathrm{B}}$=[($a_1^L+b_1^L, a_2^L+b_2^L, a_3^L+b_3^L, a_4^L+b_4^L$), ($a_1^U+b_1^U, a_2^U+b_2^U, a_3^U+b_3^U, a_4^U+b_4^U$)]

(ii) Subtraction :

$$\tilde{A} \ominus \tilde{B} = [(a_1^L - b_4^L, a_2^L - b_3^L, a_3^L - b_2^L, a_4^L - b_1^L),$$

$$(a_1^U - b_4^U, a_2^U - b_3^U, a_3^U - b_2^U, a_4^U - b_1^U)]$$

(iii) Multiplication:

$$\tilde{A} \otimes \tilde{B} = \begin{bmatrix} \left(a_1^L.\mathcal{R}(\tilde{B}), a_2^L.\mathcal{R}(\tilde{B}), a_3^L.\mathcal{R}(\tilde{B}), a_4^L.\mathcal{R}(\tilde{B})\right), \\ \left(a_1^U.\mathcal{R}(\tilde{B}), a_2^U.\mathcal{R}(\tilde{B}), a_3^U.\mathcal{R}(\tilde{B}), a_4^U.\mathcal{R}(\tilde{B})\right) \end{bmatrix} if\, \mathcal{R}(\tilde{B}) \geq 0$$

$$\tilde{A} \otimes \tilde{B} = \begin{bmatrix} \left(a_4^L.\mathcal{R}(\tilde{B}), a_3^L.\mathcal{R}(\tilde{B}), a_2^L\mathcal{R}(\tilde{B}), a_1^L\mathcal{R}(\tilde{B})\right), \\ \left(a_4^U.\mathcal{R}(\tilde{B}), a_3^U.\mathcal{R}(\tilde{B}), a_2^U\mathcal{R}(\tilde{B}), a_1^U.\mathcal{R}(\tilde{B})\right) \end{bmatrix} if\, \mathcal{R}(\tilde{B}) < 0$$

(iv) Division

$$\frac{\tilde{A}}{\tilde{B}} = [1/\mathcal{R}(\tilde{B})(a_1^L, a_2^L, a_3^L, a_4^L)\ , 1/\mathcal{R}(\tilde{B})(a_1^U, a_2^U, a_3^U, a_4^U)], if\, \mathcal{R}(\tilde{B}) > 0$$

$$= [1/\mathcal{R}(\tilde{B})(a_4^L, a_3^L, a_2^L, a_1^L)\ , 1/\mathcal{R}(\tilde{B})(a_4^U, a_3^U, a_2^U, a_1^U)]\ if\, \mathcal{R}(\tilde{B}) < 0$$

(v) Scalar Multiplication:

If $k \geq 0$ and $k \epsilon R$, then

$$k\tilde{A} = [(ka_1^L, ka_2^L, ka_3^L, ka_4^L), (ka_1^U, ka_2^U, ka_3^U, ka_4^U)]$$

if $k < 0$ and $k \epsilon R$ then

$$k\tilde{A} = [(ka_4^L, ka_3^L, ka_2^L, ka_1^L), (ka_4^U, ka_3^U, ka_2^U, ka_1^U)]$$

Definition.2.10 [6]

The Distance between any two IVFNS $\tilde{A}$ and $\tilde{B}$ can be

defined as:

$$D(\tilde{A}, B) = \frac{1}{4}\max\{|(a_1^L + a_1^U) - (b_1^L + b_1^U)| + |(a_4^L + a_4^U) - (b_4^L + b_4^U)|,$$

$$|(a_2^L + a_2^U) - (b_2^L + b_2^U)| + |(a_3^L + a_3^U) - (b_3^L + b_3^U)|\}$$

III. A DISTINCT ALGORITHM TO FIND THE CRITICAL PATH AND TOTAL FOLAT

In this Section, we define a new function called $PLMN_i$ to find the critical path of a project network.

3.1.A DISTINCT ALGORITHM FOR FINDING THE CRITICAL PATH.

STEP 1: For $\tilde{A} = [(a_1^L, a_2^L, a_3^L, a_4^L), (a_1^U, a_2^U, a_3^U, a_4^U)]$ be an interval valued fuzzy numbers, Assign the fuzzy activity time of each activity and construct the project network.

STEP 2: Find all the possible paths P_i from the network and also calculate the length of the paths PL_i

STEP 3: Find the minimum path length PL_{min}, where PL_{min} defined as follows:

If $PL^A = [(a_1^L, a_2^L, a_3^L, a_4^L), (a_1^U, a_2^U, a_3^U, a_4^U)]$ and

$PL^B = [(b_1^L, b_2^L, b_3^L, b_4^L), (b_1^U, b_2^U, b_3^U, b_4^U)]$ then

$$PLMN_i = [(m_1^L, m_2^L, m_3^L, m_4^L), (m_1^U, m_2^U, m_3^U, m_4^U)]$$

Where $m_1^L = \min(a_1^L, b_1^L)$, $m_1^U = \min(a_1^U, b_1^U)$

$m_2^L = \min(a_2^L, b_2^L)$, $m_2^U = \min(a_2^U, b_2^U)$

$$m_3^L = \min[max(a_3^L, b_3^L), max(a_2^L, b_2^L)]$$

$$m_3^U = \min[\max(a_3^U, b_3^U), \max(a_2^U, b_2^U)]$$

$$m_4^L = \min[\max(a_4^L, b_4^L), \max(a_3^L, b_3^L)$$

$$m_4^U = \min[\max(a_4^U, b_4^U), \max(a_3^U, b_3^U)]$$

STEP 4: Calculate the distance between PL_{min} and the length of each path PL_i.The path with the maximum distance will be the critical path of the project network.

STEP 5: Among the length of each path denoted the path with maximum length as PL_MSubtract the length of each path PL_i from PL_Mwhich is denoted by $PLMN_i$

STEP 6: Finding the ranking function value to all $PLMN_i$ which is denoted by $\mathcal{R}(PLMN_i)$. Choose the path with rank 1, assign the corresponding all $PLMN_i$ as the total float of each activity in that path.

STEP 7: Choose the path with rank 2 and assign the corresponding $PLMN_i$ as the total float of each activity in that path, discarding the activities already assigned.

STEP 8: The process continued until all the activities are assigned a total float.

IV. NUMERICAL ILLUSTRATION

Consider the following PERT network whose activities (in hours) are represented in the following diagram

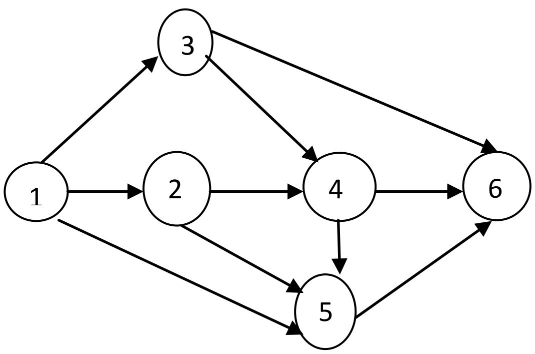

Fig 4.1 A Project Network

Table 4.1 Fuzzy activity time for each activity.

Activity A_{ij}	*Fuzzy activity times (Hours) $F\tilde{E}T_{ij}$*
A_{12}	Approximately between 2 and 3 Hours [(2,2,3,4), (1,2,3,5)]
A_{13}	Approximately between 2 and 4 Hours [(2,3,3,6), (1,2,4,7)]
A_{15}	Approximately between 3 and 4 Hours [(2,3,4,5),(1,3,4,6)]
A_{24}	Approximately between 2 and 4 Hours [(2,2,4,5),(1,2,4,6)]
A_{25}	Approximately between 4 and 5 Hours [(2,4,5,8),(1,4,5,9)]
A_{34}	Approximately between 1 and 2 Hours [(1,1,2,2),(0,1,2,3)]
A_{36}	Approximately between 7 and 12 Hours [(7,8,11,15),(6,7,12,16)]
A_{45}	Approximately between 2 and 4 Hours [(2,3,3,5),(1,2,4,6)]
A_{46}	Approximately between 3 and 4 Hours [(3,3,4,6),(2,3,4,7)]
A_{56}	Approximately around 1 Hours [(1,1,1,2),(0,1,1,3)]

Table 4.2

P_i	*PATH LENGTH PL_i*	*DISTANCE*
1-2-4-5-6	[(7,8,11,16),(3,7,12,20)]	10.75
1-2-4-6	[(7,7,11,15),(4,7,11,18)]	10.25
1-2-5-6	[(5,7,9,14),(2,7,9,17)]	7
1-3-4-5-6	[(6,8,9,15),(2,6,11,19)]	9.75
1-3-6	[(9,11,14,21),(7,9,16,23)]	14.25
1-5-6	[(3,4,5,7),(1,4,5,9)]	4.25
1-3-4-6	[(6,7,9,14),(3,6,10,17)]	6.5

Table 4.3

P_i	$PLMN_i$	$\Re(PLMN_i)$	*RANK*
1-2-4-5-6	[(-13,-7,-3,0),(-19,-8,-2,6)]	-5.75	2
1-2-4-6	[(-12,-7,-2,0),(-17,-7,-2,5)]	-5.25	3
1-2-5-6	[(-11,-5,-2,2),(-16,-5,-2,7)]	-4	6
1-3-4-5-6	[(-12,-5,-3,1),(-18,-7,-1,7)]	-4.75	4
1-3-6	[(-18,-10,-6,-2),(-22,-12,-4,2)]	-9	1
1-5-6	[(-4,-1,1,1),(-8,-1,1,8)]	0	7
1-3-4-6	[(-11,-5,-2,1),(-16,-6,-1,6)]	-4.25	5

By using the corresponding algorithm, we get the value of distance $PLMN_i$ and ranking for each possible paths.

Table 4.4

ACTIVITY	*TOTAL FLOAT*	*RANKING FUNCTION VALUE*
1-2	[(-13,-7,-3,0),(-19,-8,-2,6)]	-5.75
1-3	[(-18,-10,-6,-2),(-22,-12,-4,2)]	-9
1-5	[(-4,-1,1,4),(-8,-1,1,8)]	0
2-4	[(-13,-7,-3,0),(-19,-8,-2,6)]	-5.75
2-5	[(-11,-5,-2,2),(-16,-5,-2,7)]	-4
3-4	[(-12,-5,-3,1),(-18,-7,-1,7)]	-4.75
3-6	[(-18,-10,-6,-2),(-22,-12,-4,2)]	-9
4-5	[(-13,-7,-3,0),(-19,-8,-2,6)]	-5.75
4-6	[(-12,-7,-2,0),(-17,-7,-2,5)]	-5.25
5-6	[(-13,-7,-3,0),(-19,-8,-2,6)]	-5.75

By using the required algorithm , we get the value of total float and ranking function value for all the possible activities

Minimum ranking function value is -9 that activities are 1-3, 3-6.Hence the critical path is 1-3-6.

V. CONCLUSION

In this paper, a new distinct method has been extended to find the critical path for the project scheduling problem under the fuzzy environment. The IVFNS have been employed with new arithmetic operations in this work. New algorithmic approaches have also been introduced for finding the critical path of the problem.

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