

SMARANDACHE –BOOLEAN – NEAR –RINGS AND ALGORITHMS WITH EXAMPLES

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ABSTRACT

In this paper we introduced Smarandache-2-algebraic structure of Boolean-near-ring namely Smarandache-Boolean-near-ring. A Smarandache-2-algebraic structure on a set N means a weak algebraic structure A_0 on N such that there exists a proper subset M of N , which is embedded with a stronger algebraic structure A_1 , stronger algebraic structure means satisfying more axioms, by proper subset one understands a subset different from the empty set, form the unit element if any, from the whole set. We define Smarandache-Boolean-near-ring and obtain some of its algorithms through Boolean-ring with left-ideals, direct summand, Boolean- l -algebra, Brouwerian algebra and Compatibility. We refer to G. Pilz.

The study of Boolean-near-ring is one of the generalized structure of rings. The study and research on near-rings is very systematic and continuous. Near-rings newsletters containing complete and updated bibliography on the subject of near-rings are published periodically by a team of editors. Then motivated by several researchers we wish to study and analyse the substructure in Smarandache-near-rings. The substructure in near-rings play vital role in the study of near-rings. Unlike other algebraic structure we see in case of near-rings we have the substructure playing vital role in the study and analyse of near-rings. Apart from the sub near-rings and ideals of near-rings we have special substructure like N -groups, filter and modularity in near-rings. It is these study in the context of Smarandache-Boolean-near-rings will yield several interesting results. Also the Smarandache substructure in Boolean-near-rings will also yield very many results in the direction.

For the study we would be using the book of Pilz Gunter, Near-rings (1997) published by North Holland Press, Amsterdam [10], Special Algebraic Structure by FlorentinSmarandache, University of New Mexico, USA (1991) [18], Smarandache Algebraic Structure by Raul Padilla, Universidade do Minho, Portugal (1999) [13], Blackett [3] discusses the near-ring of affine transformations on a vector space where the near-ring has a unique maximal ideal. Gonshor [8] defines abstract of affine near-rings and completely determines the lattice of ideals for these near-rings. The near-rings of differential transformations is seen in [4]. For near-rings with geometric interpretation [10] or [18] and several research papers on Boolean-near-rings. We would first study and characterize the ideals and sub Boolean-near-rings in Smarandache-Boolean-near-rings. Also to study and analyse those Boolean-near-rings, which are Smarandache-Boolean-near-ring and find the conditions for Smarandache-Boolean-near-rings. Yet another major substructure in Boolean-near-rings is the notion of filters. We would extend and study the notion of Smarandache filters given in Smarandache-Boolean-near-rings.

Further the notion of Smarandache ideals in near-ring would be studied, characterized and analysed for Smarandache-Boolean-near-rings. Both the notions viz. N -groups and ideals in near-ring and Smarandache-boolean-near-rings would be compared and contrasted. Also the nice notion of modularity in near-rings, which are basically built using concepts of idempotents, will be studied and analysed in Smarandache modularity in Boolean-near-ring. Finally, Smarandache-Boolean-near-rings has constructed from

Boolean-ring by an algorithmic approach through its substructures and Smarandache-Boolean-near-ring has introduced some application.

Keywords: Boolean-ring, Boolean-near-ring, S -Boolean-near-ring, Boolean- l -algebra, Brouwerian algebra and Compatibility.

1. INTRODUCTION

In order that New notions are introduced in algebra to better study the congruence in number theory by FlorentinSmarandache [4]. By \langle proper subset \rangle of a set A we consider a set P included in A , and different from A , different from the empty set, and from the unit element in A – if any they rank the algebraic structures using an order relationship:

They say that the algebraic structures $S_1 \ll S_2$ if: both are defined on the same set; all S_1 laws are also S_2 laws; all axioms of an S_1 law are accomplished by the corresponding S_2 law; S_2 law accomplish strictly more axioms than S_1 laws, or S_2 has more laws than S_1 .

For example: Semi group \ll Monoid \ll group \ll ring \ll field, or Semi group \ll commutative semi group, ring \ll unitary, ring etc. They define a General special structure to be a structure SM on a set A , different from a structure SN , such that a proper subset of A is a structure, where $SM \ll SN \ll$.

2. PRELIMINARIES

DEFINITION: 2.1

A left near-ring A is a system with two binary operations, addition and multiplication, such that

- (i) the elements of A form a group $(A,+)$ under addition,
- (ii) the elements of A form a multiplicative semi-group,
- (iii) $x(y+z) = xy + xz$, for all $x,y,z \in A$

In particular, if A contains a multiplicative semi-group S whose elements generate $(A,+)$ and satisfy

- (iv) $(x+y)s = xs + ys$, for all $x, y \in A$ and $s \in S$, then we say that A is a distributively generated near-ring.

DEFINITION: 2.2

A near-ring $(B, +, \bullet)$ is Boolean-Near-Ring if there exists a Boolean-ring $(A, +, \wedge, 1)$ with identity such that \bullet is defined in terms of $+$, \wedge and 1 , and for any $b \in B$, $b \bullet b = b$

DEFINITION: 2.3

A near-ring $(B, +, \bullet)$ is said to be idempotent if $x^2 = x$, for all $x \in B$. If $(B, +, \bullet)$ is an idempotent ring, then for all $a, b \in B$, $a + a = 0$ and $a \bullet b = b \bullet a$

DEFINITION: 2.4

A Boolean-near-ring $(B, +, \bullet)$ is said to be Smarandache-Boolean-near-ring whose proper subset A is a Boolean-ring with respect to same induced operation of B .

DEFINITION: 2.5 (Alternative definition for S-Boolean-near-ring)

If there exists a non-empty set A which is a Boolean-ring such that its superset B of A is a Boolean-near-ring with respect to the same induced operation, then B is called Smarandache-Boolean-near-ring.

It can also be written as S-Boolean-near-ring.

EXAMPLE FOR SMARANDACHE-BOOLEAN-NEAR-RING: 2.6

Boolean-ring:

A Boolean-ring is an algebraic structure $(A, +, \bullet)$ together with two binary operations addition and multiplication defined as follows

$(A, +)$ is a group,

For, (i) Closure under addition :

For all $a, b \in A$, then $a + b \in A$

(ii) Associativity under addition :

For all $a, b, c \in A$, then $(a + b) + c = a + (b + c) \in A$

(iii) Commutativity of addition :

For all $a, b \in A$, then $a + b = b + a \in A$

(iv) Identity element for addition :

For all $a \in A$, then there exists 0 in A such that $0 + a = a + 0 = a \in A$

(v) Characteristic 2 for addition :

For all $a \in A$, then $a + a = 0 \in A$

(A, \bullet) is a semigroup :

(vi) Closure under product :

For all $a, b \in A$, then $a \bullet b \in A$

(vii) Associativity of product :

For all $a, b, c \in A$, then $(a \bullet b) \bullet c = a \bullet (b \bullet c) \in A$

(viii) Identity element for product :

For all a in R then there exists 1 in R such that $1 \bullet a = a \bullet 1 = a \in A$

(ix) Idempotent of product : For all $a \in A$, then $a \bullet a = a \in A$

Product is distributive over addition :

(x) Left-distributive law holds: For all $a, b, c \in A$, then $a \bullet (b + c) = (a \bullet b) + (a \bullet c) \in A$

Example for Boolean-ring :

Let $A = \{0, a\} \subset B$, be a finite-Boolean-ring. Defined by

+	0	a
0	0	a
a	a	0

•	0	a
0	0	0
a	0	a

Boolean-near-ring :

A near-ring $(B, +, \bullet)$ is said to be Boolean-near-ring if there exists a Boolean-ring $(A, +, \wedge, 1)$ with that \bullet interms $+$, \wedge and 1 , and for any $b \in B$,

$b \bullet b = b$

+	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

•	0	a	b	c
0	0	a	b	c
a	a	a	a	a
b	b	b	b	b
c	c	c	c	c

Therefore, the above conditions are satisfied, then we write B is a Boolean-near-ring.

Smarandache-Boolean-near-ring :

A Boolean-near-ring $(B, +, \bullet)$ is said to be Smarandache-Boolean-near-ring whose proper subset A is a Boolean-ring with respect to the same induced operation of B .

Verify that, B is a Smarandache-Boolean-near-ring under '+' and '•',

For check the following conditions,

$(B, +)$ is a group,

For, (i) Closure under addition :

For all $a, b \in B$, then $a + b \in B$

(ii) Associativity under addition :

For all $a, b, c \in B$, then $(a + b) + c =$

$a + (b + c) \in B$

(iii) Commutativity of addition :

For all $a, b \in B$, then $a + b = b + a \in B$

(iv) Identity element for addition :

For all $a \in B$, then there exists 0 in B such that $0 + a = a + 0 = a \in B$

(v) Characteristic 2 for addition :

For all $a \in B$, then $a + a = 0 \in B$

(B, \bullet) is a semigroup :

(vi) Closure under product :

For all $a, b \in B$, then $a \bullet b \in B$

(vii) Associativity of product :

For all $a, b, c \in B$, then $(a \bullet b) \bullet c = a \bullet (b \bullet c) \in B$

(viii) Identity element for product :

For all a in R then there exists 1 in R such that $1 \bullet a = a \bullet 1 = a \in B$

(ix) Idempotent of product :

For all $a \in A$, then $a \bullet a = a \in B$

Product is distributive over addition :

(x) Left-distributive law holds, for all $a, b, c \in B$, then $a \bullet (b + c) = (a \bullet b) + (a \bullet c) \in B$

Here A is satisfied idempotent condition of B , then we write A is a Boolean-ring, for a in A then $a \bullet a = a \in A$

Hence, the proper subset of Boolean-near-ring is a Boolean-ring and therefore, B is a Smarandache-Boolean-near-ring.

3. ALGORITHMIC STRUCTURE OF SMARNADACHE-BOOLEAN-NEAR-RING:

In this section, Algorithms to construct the S-Boolean-near-rings from its characterization are obtained.

THEOREM : 3.1

If a non-empty set B contains a unique minimal Boolean- l -algebra contained in all other non-zero Boolean- l -algebras. Then B is a Smarandache-Boolean-near-ring.

proof : Consider a Boolean-ring $I_0 \neq \{0\}$, since a Boolean-ring itself is a Boolean- l -algebra. Then I_0 is a Boolean- l -algebra. Let B be non-empty set in which A is a proper subset. Now to find subsets of B which contains I_0 such that they are Boolean- l -algebras with respect to the same induced operations of A . In Gunter Pilz [] in section 1.60. The Theorem by Gratzner and Fain is given by the following conditions for a Boolean-near-ring $B \neq \{0\}$ are equivalent

$$1. \quad \cap I \neq \{0\}, \{0\} \neq I \subseteq N$$

2. B contains a unique minimal Boolean- l -algebra, contained in all other non-zero Boolean- l -algebras.

Hence, consider the non-empty set B is a Boolean-near-ring. Now by Theorem [1], B is a Smarandache-Boolean-near-ring.

By similar argument by the Theorem[1], We have the following results

THEOREM : 3.2

If a non-empty set B contains a unique minimal Browverian algebra contained in all other non-zero Browverian algebras. Then B is a Smarandache-Boolean-near-ring.

Since in a Boolean-ring, Browverian algebras are Boolean- l -algebras. Hence by Theorem [4] gives

THEOREM : 3.3

If a non-empty set B contains a unique minimal compatibility contained in all other non-zero compatibilities. Then B is a Smarandache-Boolean-near-ring. Also by theorem [1], we have the following results

ALGORITHM : 3.1.1

BOOLEAN- l -ALGEBRA

Step 1 : Consider a Boolean-ring A

Step 2 : Verify that A is a Boolean-ring with respect to same induced operations

For, Check the following conditions, '+' is defined as follows,

1. For all $n_1, n_2 \in A$, then $n_1 + n_2 \in A$
2. For all $n_1, n_2, n_3 \in A$, then $n_1 + (n_2 + n_3) = (n_1 + n_2) + n_3$
3. For all $n \in A$, there exist $e \in A \Rightarrow n + e = e + n = n$
4. For all $n \in A$, there exist $n' \in A \Rightarrow n + n' = n' + n = e$

Let $A^* = A / \{0\}$

5. For all $n_1 \in A^* \Rightarrow n_1 \bullet n_1 = n_1 \in A^*$,
6. For all $n_1, n_2, n_3 \in A^*$, then $n_1 \bullet (n_2 \bullet n_3) = (n_1 \bullet n_2) \bullet n_3$
7. For all $n \in A^*$, there exist $e \in A \Rightarrow n \bullet e' = e' \bullet n = n$
8. For all $n \in A^*$, there exist $n' \in A \Rightarrow n \bullet n' = n' \bullet n = e'$

9. For all $n \in A^*$, then $n \bullet n = n$
10. For all $n, m \in A^*$, then $n \bullet m = m \bullet n$
11. For all $n_1, n_2, n_3 \in A^*$, then $n_1 + (n_2 \bullet n_3) = (n_1 + n_2) \bullet (n_1 + n_3)$
12. For all $n_1 \in A \Rightarrow n_1 + n_1 = 0$

The above conditions are satisfied, then we write $(B, +, \bullet)$ is a Boolean-ring.

Step 3 : Let $I_i, i = 0, 1, 2, 3$ be supersets of I_0 .

Step 4 : Let $B = \cup I_i$

Step 5 : Choose the sets I_j from I_i 's subject to for all $i, j_1, j_2 \in I_j$ such that $i \leq j_1 \leq j_2$ implies $i \cap (j_1 - j_2) = 0$

Step 6 : Verify that $\cap I_j = I_0 \neq \{0\}$

Step 7 : If step (6) is true, then we write B is a Smarandache-Boolean-near-ring.

EXAMPLE :BOOLEAN-I-ALGEBRA 3.1.2

Step 1 : Consider a non-empty set $A = \{0, n_1\}$

Step 2 : Verify that $A = \{0, n_1\}$ is a Boolean-ring with respect to same induced operations

For, Check the following conditions,
'+' is defined as follows,

$$0+0=0, \quad 0+n_1=n_1, \quad n_1+0=n_1, \quad n_1+n_1=0$$

- (i) Closure law : For all $0, n_1 \in A$,
 $0+0=0 \in A, \quad 0+n_1=n_1 \in A,$
 $n_1+0=n_1 \in A, \quad n_1+n_1=0 \in A$

(ii) Associative law : For all $0, n_1 \in A$,
 $0+(0+0) = (0+0)+0/n_1+(0+0) = (n_1+0)+0$
 $0+0 = 0+0 \quad / \quad n_1+0 = n_1+0$
 $0 = 0/n_1 = n_1$

$$0+(0+n_1) = (0+0)+n_1/n_1+(0+n_1) = (n_1+0)+n_1$$

$$0+n_1 = 0+n_1 \quad / \quad n_1+n_1 = n_1+n_1$$

$$n_1 = n_1/0=0$$

$$0+(n_1+0) = (0+n_1)+0 \quad / \quad n_1+(n_1+0) = (n_1+n_1)+0$$

$$0+n_1 = n_1+0 \quad / \quad n_1+n_1 = 0+0$$

$$n_1 = n_1 \quad 0 = 0$$

$$0+(n_1+n_1) = (0+n_1)+n_1/n_1+(n_1+n_1) = (n_1+n_1)+n_1$$

$$0+0 = n_1+n_1 \quad / \quad n_1+0 = 0+n_1$$

$$0 = 0 \quad / \quad n_1 = n_1$$

- (iii) '0' is the additive identity element : For all $0, n_1 \in A$
 $0+n_1 = n_1+0 = n_1 \quad / \quad n_1+0 = 0+n_1 = n_1$

(iv) The inverses of $0, n_1$ are respectively $0, n_1$ in A under addition.

(v) Commutative law : For all $0, n_1 \in A$

$$0+0=0+0 \quad / \quad 0+n_1=n_1+0$$

$$0 = 0 \quad / \quad n_1 = n_1$$

$$n_1+0=0+n_1 \quad / \quad n_1+n_1=n_1+n_1$$

$$n_1 = n_1 \quad / \quad 0 = 0$$

Therefore, the commutativity satisfied under addition.

(vi) Characteristic 2 for addition is defined as,

$$\text{For all } 0, n_1 \in A, \quad 0+0=0 \quad / \quad n_1+n_1=0$$

Now ' \cap ' is defined as follows :

$$\text{If } n_i \leq n_j \text{ then } n_i \cap n_j = n_i \text{ and } n_i \cap n_i = n_i,$$

$$0 \cap 0 = 0, \quad 0 \cap n_1 = 0, \quad n_1 \cap 0 = 0, \quad n_1 \cap n_1 = n_1,$$

n_1 ,

(vii) Closure law : For all $0, n_1 \in A$,

$$0 \cap 0 = 0, \quad n_1 \cap n_1 = n_1,$$

(viii) Associative law : For all $0, n_1 \in A$,

$$0 \cap (0 \cap 0) = (0 \cap 0) \cap 0 /$$

$$0 \cap 0 = 0 \cap 0$$

$$0 = 0$$

$$n_1 \cap (n_1 \cap n_1) = (n_1 \cap n_1) \cap n_1$$

$$n_1 \cap n_1 = n_1 \cap n_1$$

$$n_1 = n_1$$

Similar way, we proceed another associate laws.

(ix) Commutative law: For all $0, n_1 \in A$,

$$0 \cap 0 = 0 \cap 0 \quad / \quad n_1 \cap n_1 = n_1 \cap n_1 \quad / \quad 0 \cap n_1 = n_1 \cap 0,$$

$$0 = 0 \quad / \quad n_1 = n_1 \quad / \quad 0 = 0$$

$$n_1 \cap 0 = 0 \cap n_1 = 0 = 0$$

(x) Idempotent law : For all $0, n_1 \in A$

$$0 \cap 0 = 0 \quad \text{and} \quad n_1 \cap n_1 = n_1$$

(xi) $0 \cap (0 \cup 0) = (0 \cap 0) \cup (0 \cap 0)$

Similar way, the right distributive law holds
Therefore, $(A, +, \cap)$ is a Boolean-ring.

Step 3 : Let $A = A_0 = I_0$

$$\text{Let } I_0 = A = \{0, n_1\}$$

Step 4 : Consider the supersets $I_i, i = 0, 1, 2, 3$ of I_0 .

$$I_0 = \{0, n_1\}$$

$$I_1 = \{0, n_1, n_2\}$$

$$I_2 = \{0, n_1, n_3\}$$

$$I_3 = \{0, n_1, n_2, n_3\}$$

Step 5 : Let $B = \cup_{i=0}^3 I_i$

Step 6 : Choose set I_j 's from I_i 's subject to $n_i \leq n_j$, implies $n_i \cap (n_j - n_i) = 0$, for all $n_i, n_j \in I_j$

Step 7 : $(B, +, -, \cap)$ is defined as follows :

$+$ and \cap is defined as follows,

'+' is defined by

$$0+0=0, \quad 0+n_1=n_1, \quad 0+n_2=n_2, \quad 0+n_3=n_3,$$

$$n_1+0=n_1, \quad n_1+n_1=0, \quad n_1+n_2=n_3, \quad n_1+n_3=n_2,$$

$$n_2+0=n_2, \quad n_2+n_1=n_3, \quad n_2+n_2=0, \quad n_2+n_3=n_1,$$

$$n_3+0=n_3, \quad n_3+n_1=n_2, \quad n_3+n_2=n_1, \quad n_3+n_3=0.$$

' \cap ' is defined by, $n_i \leq n_j$ implies $n_i \cap n_j = n_i$ and $n_i \cap n_i = n_i$,

$$0 \cap 0 = 0, \quad 0 \cap n_1 = 0, \quad 0 \cap n_2 = 0, \quad 0 \cap n_3 = 0,$$

$$n_1 \cap 0 = 0, \quad n_1 \cap n_1 = n_1, \quad n_1 \cap n_2 = n_1, \quad n_1 \cap n_3 = n_1,$$

$$n_2 \cap 0 = 0, \quad n_2 \cap n_1 = n_2, \quad n_2 \cap n_2 = n_2, \quad n_2 \cap n_3 = n_2,$$

$$n_3 \cap 0 = 0, \quad n_3 \cap n_1 = n_3, \quad n_3 \cap n_2 = n_3, \quad n_3 \cap n_3 = n_3,$$

a-b is defined by $a-b = a+(a \cap b)$

For all $0 \in B, n_1 \in I_0$ then

$$\begin{aligned} 0 \cap (n_1 - 0) &= 0 \cap (n_1 + n_1 \cap 0) / \\ &= 0 \cap (n_1 + 0) \\ &= 0 \cap n_1 \\ &= 0 \in I_0 \end{aligned}$$

For all $n_1 \in B, 0 \in I_0$ then

$$\begin{aligned} n_1 \cap (0 - n_1) &= n_1 \cap (0 + 0 \cap n_1) \\ &= n_1 \cap (0 + 0) = n_1 \cap 0 \\ &= 0 \in I_0 \end{aligned}$$

For all $n_2, 0 \in B$, then,

$$\begin{aligned} n_2 \cap (0 - n_2) &= n_2 \cap (0 + 0 \cap n_2) \\ &= n_2 \cap (0 + 0) \\ &= n_2 \cap 0 \\ &= 0 \in I_0 \end{aligned}$$

For all $n_3 \in B, 0 \in I_0$ then,

$$\begin{aligned} n_3 \cap (0 - n_3) &= n_3 \cap (0 + 0 \cap n_3) = n_3 \cap (0 + 0) \\ &= n_3 \cap 0 \\ &= 0 \in I_0 \end{aligned}$$

For all $n_2 \in B, n_1 \in I_0$ then,

$$\begin{aligned} n_2 \cap (n_1 - n_2) &= n_2 \cap (n_1 + n_1 \cap n_2) = n_2 \cap (n_1 + n_1) \\ &= n_2 \cap 0 \\ &= 0 \in I_0 \end{aligned}$$

For all $n_3 \in B, n_1 \in I_0$ then,

$$\begin{aligned} n_3 \cap (n_1 - n_3) &= n_3 \cap (n_1 + n_1 \cap n_3) \\ &= n_3 \cap (n_1 + n_1) = n_3 \cap 0 \\ &= 0 \in I_0 \end{aligned}$$

For all $n_1 \in B, n_1 \in I_0$ then /

$$\begin{aligned} n_1 \cap (n_1 - n_1) &= n_1 \cap (n_1 + n_1 \cap n_1) \\ &= n_1 \cap (n_1 + n_1) \\ &= n_1 \cap 0 = 0 \in I_0 \end{aligned}$$

For all $0 \in B, 0 \in I_0$ then,

$$\begin{aligned} 0 \cap (0 - 0) &= 0 \cap (0 + 0 \cap 0) \\ &= 0 \cap (0 + 0) = 0 \cap 0 \\ &= 0 \in I_0 \end{aligned}$$

Hence, I_0 satisfies all the requirements.

Therefore, we choose I_0 as I_j .

Hence for $I_1 = \{0, n_1, n_2\}$ and $B = \{0, n_1, n_2, n_3\}$

$$n_i \cap (n_j - n_i) = n_i \cap (n_j + n_j \cap n_i) = 0, \text{ for all } n_i, n_j \in B, \text{ and } 0 \in I_1.$$

Therefore, I_1 also choose as I_j .

Similarly, for $I_2 = \{0, n_1, n_3\}$ and $B = \{0, n_1, n_2, n_3\}$

$$n_i \cap (n_j - n_i) = n_i \cap (n_j + n_j \cap n_i) = 0, \text{ for all } n_i, n_j \in B, \text{ and } 0 \in I_2.$$

Therefore, I_2 also choose as I_j .

Similarly, for $I_3 = \{0, n_2, n_3\}$ and $B = \{0, n_1, n_2, n_3\}$

$$n_i \cap (n_j - n_i) = n_i \cap (n_j + n_j \cap n_i) = 0, \text{ for all } n_i, n_j \in B, \text{ and } 0 \in I_3.$$

Therefore, I_3 becomes I_j .

Step 8 : Verify that $\bigcap I_j = I_0 \neq \{0\} \subset B$

$$\begin{aligned} I_0 \cap I_1 \cap I_2 \cap I_3 &= \{0, n_1\} \cap \{0, n_1, n_2\} \cap \{0, n_1, n_3\} \cap \{0, n_2, n_3\} \\ &= \{0, n_1\} \neq \{0\} \subset B \end{aligned}$$

Step 9 : If step (8) is true, then we write B is a Smarandache-Boolean-near-ring.

ALGORITHM : 3.2.1

BROWERIAN ALGEBRA

Step 1 : Consider a Boolean-ring A

Step 2 : Verify that A is a Boolean-ring with respect to same induced operations

For, Check the following conditions,

'+' is defined as follows,

1. For all $n_1, n_2 \in A$, then $n_1 + n_2 \in A$

2. For all $n_1, n_2, n_3 \in A$, then $n_1 + (n_2 + n_3) = (n_1 + n_2) + n_3$

3. For all $n \in A$, there exist $e \in A \Rightarrow n + e = e + n = n$

4. For all $n \in A$, there exist $n' \in A \Rightarrow n + n' = n' + n = e$

Let $A^* = A / \{0\}$

5. For all $n_1 \in A^* \Rightarrow n_1 \bullet n_1 = n_1 \in A^*$,

6. For all $n_1, n_2, n_3 \in A^*$, then $n_1 \bullet (n_2 \bullet n_3) = (n_1 \bullet n_2) \bullet n_3$

7. For all $n \in A^*$, there exist $e \in A \Rightarrow n \bullet e' = e' \bullet n = n$

8. For all $n \in A^*$, there exist $n' \in A \Rightarrow n \bullet n' = n' \bullet n = e'$

9. For all $n \in A^*$, then $n \bullet n = n$

10. For all $n, m \in A^*$, then $n \bullet m = m \bullet n$

11. For all $n_1, n_2, n_3 \in A^*$, then $n_1 + (n_2 \bullet n_3) = (n_1 + n_2) \bullet (n_1 + n_3)$

12. For all $n_1 \in A \Rightarrow n_1 + n_1 = 0$

The above conditions are satisfied, then write $(B, +, \bullet)$ is a Boolean-ring.

Step 3 : Let $I_i, i = 0, 1, 2, 3, \dots$ be the supersets of I_0 .

Step 4 : Let $B = \bigcup I_i$

Step 5 : Choose the sets I_j from I_i 's subject to for all $x \in B$ such that $x \leq a$ then $a = x \cup (a-x)$, for all x and $a \in I_j$

Step 6 : Verify that $\bigcap I_j = I_0 \neq \{0\}$

Step 7 : If step (6) is true, then we write B is a Smarandache-Boolean-near-ring.

EXAMPLE : BROWERIAN-ALGEBRA 3.2.2

Step 1 : Consider a non-empty set $A = \{0, n_1\}$

Step 2 : Verify that $A = \{0, n_1\}$ is a Boolean-ring with respect to same induced operations,

For, Check the following conditions,

'+' is defined as follows,

$0+0=0, 0+n_1=n_1, n_1+0=n_1, n_1+n_1=0$

(i) Closure law : For all $0, n_1 \in A$,

$$0+0 = 0 \in A, 0+n_1 = n_1 \in A, n_1+0 = n_1 \in A, n_1+n_1 = 0 \in A$$

(ii) Associative law : For all $0, n_1 \in A$,

$$\begin{aligned} 0+(0+0) &= (0+0)+0 & / & n_1+(0+0) = (n_1+0)+0 \\ 0+0 &= 0+0 & / & n_1+0 = n_1+0 \\ 0 &= 0 & / & n_1 = n_1 \end{aligned}$$

$$\begin{aligned} 0+(0+n_1) &= (0+0)+n_1 & / & n_1+(0+n_1) = (n_1+0)+n_1 \\ 0+n_1 &= 0+n_1 & / & n_1+n_1 = n_1+n_1 \\ n_1 &= n_1 & / & 0 = 0 \end{aligned}$$

$$\begin{aligned} 0+(n_1+0) &= (0+n_1)+0 & / & n_1+(n_1+0) = (n_1+n_1)+0 \\ 0+n_1 &= n_1+0 & / & n_1+n_1 = 0+0 \\ 0 &= 0 & / & n_1 = n_1 \end{aligned}$$

$$\begin{aligned} 0+(n_1+n_1) &= (0+n_1)+n_1 & / & n_1+(n_1+n_1) = (n_1+n_1)+n_1 \\ 0+0 &= n_1+n_1 & / & n_1+0 = 0+n_1 \\ 0 &= 0 & / & n_1 = n_1 \end{aligned}$$

(iii) '0' is the additive identity element :

$$\begin{aligned} \text{For all } 0, n_1 \in A \\ 0+n_1 = n_1+0 = n_1 & \quad / \quad n_1+0 = 0+n_1 = n_1 \end{aligned}$$

(iv) The inverses of $0, n_1$ are respectively $0, n_1$ in A under addition.

(v) Commutative law : For all $0, n_1 \in A$

$$\begin{aligned} 0+0 &= 0+0 & / & 0+n_1 = n_1+0 \\ 0 &= 0 & / & n_1 = n_1 \end{aligned}$$

$$\begin{aligned} n_1+0 &= 0+n_1 & / & n_1+n_1 = n_1+n_1 \\ n_1 &= n_1 & / & 0 = 0 \end{aligned}$$

Therefore, the commutative law satisfied under addition.

(vi) Characteristic 2 for addition is defined as,

$$\begin{aligned} \text{For all } 0, n_1 \in A, \\ 0+0 = 0 & \quad / \quad n_1+n_1 = 0 \end{aligned}$$

Now ' \cap ' is defined as follows :

If $n_i \leq n_j$ then $n_i \cap n_j = n_i$ and $n_i \cap n_i = n_i$,

$$0 \cap 0 = 0, 0 \cap n_1 = 0, n_1 \cap 0 = 0, n_1 \cap n_1 = n_1,$$

(vii) Closure law : For all $0, n_1 \in A$,

$$0 \cap 0 = 0, n_1 \cap n_1 = n_1,$$

(viii) Associative law : For all $0, n_1 \in A$,

$$\begin{aligned} 0 \cap (0 \cap 0) &= (0 \cap 0) \cap 0 & / & \\ 0 \cap 0 &= 0 \cap 0 & / & \\ 0 &= 0 & / & \\ n_1 \cap (n_1 \cap n_1) &= (n_1 \cap n_1) \cap n_1 & / & \\ n_1 \cap n_1 &= n_1 \cap n_1 & / & \\ n_1 &= n_1 & / & \end{aligned}$$

Similar way, we proceed another associate laws.

(ix) Commutative law: For all $0, n_1 \in A$,

$$\begin{aligned} 0 \cap 0 &= 0 \cap 0 & / & n_1 \cap n_1 = n_1 \cap n_1 & / & 0 \cap n_1 = n_1 \cap 0 & / & \\ 0 &= 0 & / & n_1 &= n_1 & / & 0 &= 0 & / & \\ n_1 \cap 0 &= 0 \cap n_1 &= & 0 &= 0 & & & & & \end{aligned}$$

(x) Idempotent law : For all $0, n_1 \in A$

$$0 \cap 0 = 0 \text{ and } n_1 \cap n_1 = n_1$$

(xi) For all $0, n_1 \in A$

$$0 \cap (0 \cup 0) = (0 \cap 0) \cup (0 \cap 0)$$

$$n_1 \cap (n_1 \cup n_1) = (n_1 \cap n_1) \cup (n_1 \cap n_1)$$

Similar way, the right distributive law holds

(xii) Define, the complement: For all $0, n_1 \in A, 0' \cap n_1 = 0$

Therefore, $(A, +, \cap, \cup)$ is a Boolean-ring.

Step 3 : Let $A = A_0 = I_0$

$$\text{Let } I_0 = A = \{0, n_1\}$$

Step 4 : Consider the supersets $I_i, i = 0, 1, 2, 3$ of I_0 .

$$I_0 = \{0, n_1\}$$

$$I_1 = \{0, n_1, n_2\}$$

$$I_2 = \{0, n_1, n_3\}$$

$$I_3 = \{0, n_1, n_2, n_3\}$$

Step 5 : Let $B = \bigcup_{i=0}^3 I_i$

Step 6 : Choose set I_j 's from I_i 's subject to $x \leq a$ implies $a = x \cup (a - x)$, for all $a, x \in I_j$

Step 7 : $(B, +, -, \cup)$ is defined as follows :

$+$ and \cup is defined as follows,

'+' is defined by

$$0+0 = 0, 0+n_1 = n_1, 0+n_2 = n_2, 0+n_3 = n_3,$$

$$n_1+0 = n_1, n_1+n_1 = 0, n_1+n_2 = n_3, n_1+n_3 = n_2,$$

$$n_2+0 = n_2, n_2+n_1 = n_3, n_2+n_2 = 0, n_2+n_3 = n_1,$$

$$n_3+0 = n_3, n_3+n_1 = n_2, n_3+n_2 = n_1, n_3+n_3 = 0.$$

' \cap ' is defined by, $n_i \leq n_j$ implies $n_i \cap n_j = n_i$ and $n_i \cap n_i = n_i$,

$$0 \cap 0 = 0, 0 \cap n_1 = 0, 0 \cap n_2 = 0, 0 \cap n_3 = 0,$$

$$n_1 \cap 0 = 0, n_1 \cap n_1 = n_1, n_1 \cap n_2 = n_1, n_1 \cap n_3 = n_1,$$

$$n_2 \cap 0 = 0, n_2 \cap n_1 = n_3, n_2 \cap n_2 = n_2, n_2 \cap n_3 = n_2,$$

$$n_3 \cap 0 = 0, n_3 \cap n_1 = n_3, n_3 \cap n_2 = n_3, n_3 \cap n_3 = n_3,$$

' \cup ' is defined by

$$0 \cup 0 = 0, 0 \cup n_1 = n_1, 0 \cup n_2 = n_2, 0 \cup n_3 = n_3,$$

$$n_1 \cup 0 = n_1, n_1 \cup n_1 = n_1, n_1 \cup n_2 = n_2, n_1 \cup n_3 = n_3,$$

$$n_2 \cup 0 = n_2, n_2 \cup n_1 = n_3, n_2 \cup n_2 = n_2, n_2 \cup n_3 = n_3,$$

$$n_3 \cup 0 = n_3, n_3 \cup n_1 = n_3, n_3 \cup n_2 = n_3, n_3 \cup n_3 = n_3,$$

$n - m$ is defined by $n - m = n$ if $n \leq m$

For all $0 \in B, n_1 \in I_0$ then / For all $n_1 \in B, 0 \in I_0$ then

$$0 \cup (n_1 - 0) = 0 \cup (n_1 + n_1 \cap 0)$$

$$= 0 \cup (n_1 + 0)$$

$$= 0 \cup n_1 = n_1 \in I_0$$

$$n_1 \cup (0 - n_1) = n_1 \cup (0 + 0 \cap n_1)$$

$$= n_1 \cup (0 + 0)$$

$$= n_1 \cup 0$$

$$= n_1 \in I_0$$

For all $n_2 \in B, 0 \in I_0$ then, /

$$0 \cup (n_2 - 0) = 0 \cup (n_2 + n_2 \cap 0)$$

$$= 0 \cup (n_2 + 0)$$

$$= 0 \cup n_2 = 0' \cap n_2' = n_3 \cap n_1$$

$$= n_1 \in I_0$$

For all $n_3 \in B, 0 \in I_0$ then,

$$n_3 \cup (0 - n_3) = n_3 \cup (0 + 0 \cap n_3)$$

$$\begin{aligned}
 &= n_3 \cup (0 + 0) \\
 &= n_3 \cup 0 = n_3 \cap 0' = 0 \cap n_3 \\
 &= 0 \in I_0
 \end{aligned}$$

For all $n_2 \in B, n_1 \in I_0$ then, /

$$\begin{aligned}
 n_2 \cup (n_1 - n_2) &= n_2 \cup (n_1 + n_1 \cap n_2) \\
 &= n_2 \cup (n_1 + n_1) \\
 &= n_2 \cup 0 \\
 &= n_2 \in I_1
 \end{aligned}$$

For all $n_3 \in B, n_1 \in I_0$ then,

$$\begin{aligned}
 n_3 \cup (n_1 - n_3) &= n_3 \cup (n_1 + n_1 \cap n_3) \\
 &= n_3 \cup (n_1 + n_1) \\
 &= n_3 \cup 0 = n_3 \cap 0' = 0 \cap n_3 \\
 &= 0 \in I_1
 \end{aligned}$$

For all $n_1 \in B, n_1 \in I_0$ then /

$$\begin{aligned}
 n_1 \cup (n_1 - n_1) &= n_1 \cup (n_1 + n_1 \cap n_1) \\
 &= n_1 \cup (n_1 + n_1) \\
 &= n_1 \cup 0 \\
 &= n_1 \in I_0
 \end{aligned}$$

For all $0 \in B, 0 \in I_0$ then,

$$\begin{aligned}
 0 \cup (0 - 0) &= 0 \cup (0 + 0 \cap 0) \\
 &= 0 \cup (0 + 0) \\
 &= 0 \cup 0 \\
 &= 0 \in I_0
 \end{aligned}$$

Hence, I_0 satisfies all the requirements.

Therefore, we choose I_0 as I_j . Hence for $I_1 = \{0, n_1, n_2\}$ and $B = \{0, n_1, n_2, n_3\}$
 $n \cup (m - n) = m$, for all $n, m \in B$, and $0 \in I_1$.
 Therefore, I_1 also choose as I_j .

Similarly, for $I_2 = \{0, n_1, n_3\}$ and $B = \{0, n_1, n_2, n_3\}$
 $n \cup (m - n) = m$, for all $n, m \in B$, and $0 \in I_2$.
 Therefore, I_2 also choose as I_j .

Similarly, for $I_3 = \{0, n_2, n_3\}$ and $B = \{0, n_1, n_2, n_3\}$
 $n \cup (m - n) = m$, for all $n, m \in B$, and $0 \in I_3$.
 Therefore, I_3 becomes I_j .

Step 8 : Verify that $\bigcap I_j = I_0 \neq \{0\} \subset B$

$$\begin{aligned}
 I_0 \cap I_1 \cap I_2 \cap I_3 &= \{0, n_1\} \cap \{0, n_1, n_2\} \cap \{0, n_1, n_3\} \cap \{0, n_1, n_2, n_3\} \\
 &= \{0, n_1\} \neq \{0\} \subset B
 \end{aligned}$$

Step 9 : If step (8) is true, then we write B is a Smarandache-Boolean-near-ring.

ALGORITHM : 3.3.1
COMPATIBILITY :

Step 1 : Consider a Boolean-ring A

Step 2 : Verify that A is a Boolean-ring with respect to same induced operations
 For, Check the following conditions,
 '+' is defined as follows,
 1. For all $n_1, n_2 \in A$, then $n_1 + n_2 \in A$

2. For all $n_1, n_2, n_3 \in A$, then $n_1 + (n_2 + n_3) = (n_1 + n_2) + n_3$

3. For all $n \in A$, there exist $e \in A \Rightarrow n + e = e + n = n$

4. For all $n \in A$, there exist $n' \in A \Rightarrow n + n' = n' + n = e$

Let $A^* = A / \{0\}$

5. For all $n_1 \in A^* \Rightarrow n_1 \bullet n_1 = n_1 \in A^*$,

6. For all $n_1, n_2, n_3 \in A^*$, then $n_1 \bullet (n_2 \bullet n_3) = (n_1 \bullet n_2) \bullet n_3$

7. For all $n \in A^*$, there exist $e \in A \Rightarrow n \bullet e' = e' \bullet n = n$

8. For all $n \in A^*$, there exist $n' \in A \Rightarrow n \bullet n' = n' \bullet n = e'$

9. For all $n \in A^*$, then $n \bullet n = n$

10. For all $n, m \in A^*$, then $n \bullet m = m \bullet n$

11. For all $n_1, n_2, n_3 \in A^*$, then $n_1 + (n_2 \bullet n_3) = (n_1 + n_2) \bullet (n_1 + n_3)$

12. For all $n_1 \in A \Rightarrow n_1 + n_1 = 0$

The above conditions are satisfied, then write $(B, +, \bullet)$ is a Boolean-ring.

Step 3 : Let $I_i, i = 0, 1, 2, 3, \dots$ be the supersets of I_0 .

Step 4 : Let $B = \bigcup I_i$

Step 5 : Choose the sets I_j from I_i 's subject to for all $a, b \in I_j$ such that $ab^2 = a^2b \in I_j$

Step 6 : Verify that $\bigcap I_j = I_0 \neq \{0\}$

Step 7 : If step (6) is true, then we write B is a Smarandache-Boolean-near-ring.

EXAMPLE : COMPATIBILITY 3.3.2

Step 1 : Consider a non-empty set $A = \{0, n_1\}$

Step 2 : Verify that $A = \{0, n_1\}$ is a Boolean-ring with respect to same induced operations

For, Check the following conditions,

'+' is defined as follows,

$$0 + 0 = 0, \quad 0 + n_1 = n_1, \quad n_1 + 0 = n_1, \quad n_1 + n_1 = 0$$

(i) Closure law : For all $0, n_1 \in A$,

$$0 + 0 = 0 \in A, \quad 0 + n_1 = n_1 \in A, \quad n_1 + 0 = n_1 \in A, \quad n_1 + n_1 = 0 \in A$$

(ii) Associative law : For all $0, n_1 \in A$,

$$0 + (0 + 0) = (0 + 0) + 0 / n_1 + (0 + 0) = (n_1 + 0) + 0$$

$$0 + 0 = 0 + 0 \quad / \quad n_1 + 0 = n_1 + 0$$

$$0 = 0 \quad / \quad n_1 = n_1$$

$$0 + (0 + n_1) = (0 + 0) + n_1 / n_1 + (0 + n_1) = (n_1 + 0) + n_1$$

$$0 + n_1 = 0 + n_1 \quad / \quad n_1 + n_1 = n_1 + n_1$$

$$n_1 = n_1 \quad / \quad 0 = 0$$

$$0 + (n_1 + 0) = (0 + n_1) + 0 / n_1 + (n_1 + 0) = (n_1 + n_1) + 0$$

$$0 + n_1 = n_1 + 0 \quad / \quad n_1 + n_1 = 0 + 0$$

$$n_1 = n_1 \quad 0 = 0$$

$$0 + (n_1 + n_1) = (0 + n_1) + n_1 / n_1 + (n_1 + n_1) = (n_1 + n_1) + n_1$$

$$0 + 0 = n_1 + n_1 \quad / \quad n_1 + 0 = 0 + n_1$$

$$0 + 0 = 0 \quad / \quad n_1 + n_1 = n_1$$

(iii) '0' is the additive identity element : For all $0, n_1 \in A$
 $0+n_1 = n_1+0 = n_1 \quad / \quad n_1+0 = 0+n_1 = n_1$

(iv) The inverses of $0, n_1$ are respectively $0, n_1$ in A under addition.

(v) Commutative law : For all $0, n_1 \in A$
 $0 + 0 = 0 + 0 \quad / \quad 0+n_1 = n_1+0$
 $0 = 0 \quad / \quad n_1 = n_1$

$$n_1+0 = 0+n_1 \quad / \quad n_1+n_1 = n_1+n_1$$

$$n_1 = n_1 \quad / \quad 0 = 0$$

Therefore, the commutativity satisfied under addition.

(vi) Characteristic 2 for addition is defined as,
 For all $0, n_1 \in A$,
 $0 + 0 = 0 \quad / \quad n_1+n_1 = 0$

Let $A^* = A / \{0\} = \{0, n_1\} = n_1$,
 Now '•' is defined as follows,
 $0 \bullet 0 = 0, 0 \bullet n_1 = 0, n_1 \bullet 0 = 0, n_1 \bullet n_1 = n_1$

In general, we define $n \bullet m = n$, for all $n, m \in A$

(vii) For all $n_1 \in A^*$, $n_1 \bullet n_1 = n_1 \in A^*$,

(viii) For all $n_1 \in A^*$,

$$n_1 \bullet (n_1 \bullet n_1) = (n_1 \bullet n_1) \bullet n_1$$

$$n_1 \bullet n_1 = n_1 \bullet n_1$$

$$n_1 = n_1$$

(ix) 'n₁' is the identity element of A^* under multiplication

(x) For all $0, n_1 \in A^*$,
 $0 \bullet 0 = 0 \quad / \quad n_1 \bullet n_1 = n_1$

Hence, the idempotent law is satisfied under multiplication.

(xi) Product is distributive over addition as follows:

For all $0, n_1 \in A$,

$$0 \bullet (0+0) = 0 \bullet 0 + 0 \bullet 0$$

$$0 \bullet 0 = 0 + 0$$

$$0 = 0$$

$$n_1 \bullet (n_1 + n_1) = (n_1 \bullet n_1) + (n_1 \bullet n_1)$$

$$n_1 \bullet 0 = n_1 + n_1$$

$$0 = 0$$

$$n_1 \bullet (0+0) = n_1 \bullet 0 + n_1 \bullet 0$$

$$n_1 \bullet 0 = 0 + 0$$

$$0 = 0$$

$$n_1 \bullet (0 + n_1) = n_1 \bullet 0 + n_1 \bullet n_1$$

$$n_1 \bullet n_1 = 0 + n_1$$

$$n_1 = n_1$$

$$n_1 \bullet (n_1 + 0) = n_1 \bullet n_1 + n_1 \bullet 0 \quad / \quad 0 \bullet (n_1 + n_1) = 0 \bullet n_1 + 0 \bullet n_1$$

$$n_1 \bullet n_1 = n_1 + 0 \quad / \quad 0 \bullet 0 = 0 + 0$$

$$n_1 = n_1 \quad / \quad 0 = 0$$

$$0 \bullet (n_1 + 0) = 0 \bullet n_1 + 0 \bullet 0 \quad / \quad 0 \bullet (0 + n_1) = 0 \bullet 0 + 0 \bullet n_1$$

$$0 \bullet n_1 = 0 + 0 \quad / \quad 0 \bullet n_1 = 0 + 0$$

$$0 = 0 \quad / \quad 0 = 0$$

Similar way, the right distributive law holds.

(xii) Commutative law : For all $0, n_1 \in A$
 $0 \bullet 0 = 0 \bullet 0 \quad / \quad 0 \bullet n_1 = n_1 \bullet 0$
 $0 = 0 \quad / \quad 0 = 0$

$$n_1 \bullet 0 = 0 \bullet n_1 \quad / \quad n_1 \bullet n_1 = n_1 \bullet n_1$$

$$0 = 0 \quad / \quad n_1 = n_1$$

Hence, all the requirements of Boolean-rings are satisfied.
 Therefore, $(A, +, \bullet)$ is a Boolean-ring.

Step 3 : Let $A = A_0 = I_0$
 Let $I_0 = A = \{0, n_1\}$

Step 4 : Consider the supersets $I_i, i = 0,1,2,3$ of I_0 .
 $I_0 = \{0, n_1\}$
 $I_1 = \{0, n_1, n_2\}$
 $I_2 = \{0, n_1, n_3\}$
 $I_3 = \{0, n_1, n_2, n_3\}$

Step 5 : Let $B = \bigcup_{i=0}^m I_i$

Step 6 : Choose set I_j 's from I_i 's subject to for all $a, b \in I_j$ such that
 $ab^2 = a^2b \in I_j$

Step 7 : $(B, +, \bullet)$ is defined as follows :
 $+$ and \bullet is defined as follows,
 '+' is defined by

$$0+0 = 0, 0+n_1 = n_1, 0+n_2 = n_2, 0+n_3 = n_3,$$

$$n_1+0 = n_1, n_1+n_1 = 0, n_1+n_2 = n_3, n_1+n_3 = n_2,$$

$$n_2+0 = n_2, n_2+n_1 = n_3, n_2+n_2 = 0, n_2+n_3 = n_1,$$

$$n_3+0 = n_3, n_3+n_1 = n_2, n_3+n_2 = n_1, n_3+n_3 = 0.$$

'•' is defined by

$$0 \bullet 0 = 0, 0 \bullet n_1 = 0, 0 \bullet n_2 = 0, 0 \bullet n_3 = 0,$$

$$n_1 \bullet 0 = n_1, n_1 \bullet n_1 = n_1, n_1 \bullet n_2 = n_1, n_1 \bullet n_3 = n_1,$$

$$n_2 \bullet 0 = n_2, n_2 \bullet n_1 = n_2, n_2 \bullet n_2 = n_2, n_2 \bullet n_3 = n_2,$$

$$n_3 \bullet 0 = n_3, n_3 \bullet n_1 = n_3, n_3 \bullet n_2 = n_3, n_3 \bullet n_3 = n_3,$$

$nm^2 = n^2m$ is defined by

For all $0 \in B, n_1 \in I_0$ then / For all $n_1 \in B, 0 \in I_0$ then

$$0 n_1^2 = 0^2 n_1 \quad / \quad n_1 0 = n_1^2 0$$

$$0 n_1 = 0 n_1 \quad / \quad n_1 0 = n_1 0$$

$$0 = 0 \quad / \quad 0 = 0$$

$$= 0 \in I_0 \quad / \quad = 0 \in I_0$$

For all $n_2, 0 \in B$, then, / For all $n_3 \in B, 0 \in I_0$ then,

$$n_1 0 = n_1^2 0 \quad / \quad n_3^2 0 = 0^2 n_3$$

$$n_1 0 = n_1 0 \quad / \quad n_3 0 = n_3^2 0$$

$$0 = 0 \quad / \quad 0 = 0$$

$$= 0 \in I_0 \quad / \quad = 0 \in I_0$$

For all $n_2 \in B, n_1 \in I_0$ then / For all $n_3 \in B, n_1 \in I_0$ then,

$$n_2 n_1^2 = n_2^2 n_1 \quad / \quad n_3 n_1^2 = n_3^2 n_1$$

$$n_2 n_1 = n_2 n_1 \quad / \quad n_3 n_1 = n_3 n_1$$

$$n_1 n_2 = n_1 n_2 \quad / \quad n_1 n_3 = n_1 n_3$$

$$n_1 = n_1 \quad / \quad n_1 = n_1$$

$$= n_1 \in I_0 \quad / \quad = n_1 \in I_0$$

For all $n_1 \in B, n_1 \in I_0$ then / For all $0 \in B, 0 \in I_0$ then,

$$n_1 n_1^2 = n_1^2 n_1 \quad / \quad 0 0^2 = 0^2 0$$

$$n_1 n_1 = n_1 n_1 \quad / \quad 0 0 = 0 0$$

$$n_1 = n_1 \quad / \quad 0 = 0$$

$$= n_1 \in I_0 \quad / \quad = 0 \in I_0$$

Hence, I_0 satisfies all the requirements.

Therefore, we choose I_0 as I_j . Hence for $I_1 = \{0, n_1, n_2\}$ and
 $B = \{0, n_1, n_2, n_3\}$

$$n_i n_j^2 = n_i^2 n_j, \text{ for all } n_i, n_j \in B, \text{ and } 0 \in I_1.$$

Therefore, I_1 also choose as I_j .

Similarly, for $I_2 = \{0, n_1, n_3\}$ and $B = \{0, n_1, n_2, n_3\}$

$$n_i n_j^2 = n_i^2 n_j, \text{ for all } n_i, n_j \in B, \text{ and } 0 \in I_2.$$

Therefore, I_2 also choose as I_j .

Similarly, for $I_3 = \{0, n_2, n_3\}$ and $B = \{0, n_1, n_2, n_3\}$

$$n_i n_j^2 = n_i^2 n_j, \text{ for all } n_i, n_j \in B, \text{ and } 0 \in I_3.$$

Therefore, I_3 becomes I_j .

Step 8 : Verify that $\bigcap I_j = I_0 \neq \{0\} \subset B$

$$\begin{aligned} I_0 \cap I_1 \cap I_2 \cap I_3 &= \{0, n_1\} \cap \{0, n_1, n_2\} \cap \{0, n_1, n_3\} \cap \{0, n_1, n_2, n_3\} \\ &= \{0, n_1\} \neq \{0\} \subset B \end{aligned}$$

Step 9 : If step (8) is true, then we write B is a Smarandache-Boolean-near-ring.

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