# Contact Stress Analysis of Involute Spur gear under Static loading

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## ABSTRACT
Gears are dominating the mechanical industries as a main transmission system for almost two centuries and it is still the most reliable and efficient systems for any transmission mechanisms. The life of gears are mainly effected due to the higher amount of stresses developed, basically the gears experienced two major stresses i.e. bending and contact stresses. In this study we have analyzed a spur gear pair from lathe machine for contact stresses under static loading conditions using Hertz theory and finite element analysis. The analysis highlights the effect of different modules on the contact stresses and finally the results of Hertz theory and Finite element analysis is compared.

***Keywords*** *– finite element analysis, gear module, Hertz theory, Involute spur gear, static loading.*

## I. INTRODUCTION
Base on the successively engaging projection called teeth, gears are direct contact bodies, operating in pair, which transmit motion and force from one rotating shaft to another, it is one of the most important mechanical transmission forms in the field of mechanical engineering, it has various forms and uses widely, but the consequences caused by gear failure are often serious. Loaded contact performance, tooth contact stress and tooth root bending stress of the pair of gears are an important concerned index in the successive process of engagement. [1] This study mainly focus on the sole contact stresses which effect the gear life to a large extent.
Gears are a critical component in the rotating machinery industry. Various research methods, such as theoretical, numerical, and experimental, have been done throughout the years regarding gears. One of the reasons why theoretical and numerical methods are preferred is because experimental testing can be particularly expensive.
Thus, numerous mathematical models of gears have been developed for different purposes [3].
In detail study of the contact stress produced in the mating gears is the most important task in design of gears as it is the deciding parameter in finding the dimensions of gear as explained by Bharat Gupta et al [4]. Also the module of a gear plays an important role in transmitting the power between two shafts. The spur gear with higher module is the best choice for transmitting large power between the parallel shafts as explained by Bharat Gupta et al [4].
The conventional method to calculate the contact stresses in the Hertz Theory which is based on contact between two cylinders. We have taken a involute spur gear pair from a lathe machine and carried out the hertz analytical calculation to find out the contact stresses. The contact stresses mainly varies as the module of the gear varies. This we have cross examined with analyzing the same gear pair using the finite element analysis, which testifies the same pattern.
The gear pair is modeled in creo2.0 design software and later exported to Ansys14.5 Mechanical modeler for static stress analysis.

## II. LITERATURE REVIEW
Ali Raad Hassan (2009) did a research study in which Contact stress analysis between two spur gear teeth was studied in different contact positions, representing a pair of mating gears during rotation. A platform has been established to plot a pair of teeth in contact. Each case was represented a sequence position of contact between these two teeth. The platform gives graphic results for the profiles of these teeth in each position and location of contact during rotation. Finite element models were made for these cases and stress analysis was done. The results were presented and finite element analysis results were compared with theoretical calculations, wherever available.
Bharat Gupta, Abhishek Choubey, Gautam V. Varde (2012) presented a paper to suggest that, thorough study of contact stress developed between the different matting gears are mostly important for the gear design. They have used Hertz equations which is a analytical method of calculating gear contact stresses, originally derived for contact between two cylinders. So for contact stress they established and determined appropriate models of contact elements, and calculated contact stresses using ANSYS and compared the results with Hertz theory. Conclusions suggest that with increase in module, contact stresses decreases for a pair of spur gears.

Sushil Kumar Tiwari, Upendra Kumar Joshi (2011) this paper presented analysis of Bending stress and Contact stress of Involute spur gear teeth in meshing. Bending stress and contact stress calculation is the basic of stress analysis. It is difficult to get correct answer on gear tooth stress by implying fundamental Hertz equation for contact stress. They primarily prefer Theoretical and Numerical methods because Experimental testing can be expensive.

Vivek Karaveer, Ashish Mogrekar and T. Preman Reynold Joseph (2013) this paper presents the stress analysis of mating teeth of spur gear to find maximum contact stress in the gear teeth. The results obtained from Finite Element Analysis (FEA) are compared with theoretical Hertzian equation values. For the analysis, steel and Grey cast iron are used as the materials of spur gear. The spur gears are sketched, modeled and assembled in ANSYS Design Modeler. As Finite Element Method (FEM) is the easy and accurate technique for stress analysis, FEA is done in finite element software ANSYS 14.5. Also deformation for steel and Grey cast iron is obtained as efficiency of the gear depends on its deformation. The results shown that the difference between maximum contact stresses obtained from Hertz equation and Finite Element Analysis is very less and it is acceptable. The deformation patterns of steel and Grey cast iron gears depicted that the difference in their deformation is negligible.

M. Raja, P. Phani"s work made an attempt to summarize about contact stresses developed in a mating spur gear which has involute teeth. A pair of spur gears are taken from a lathe gear box and proceeded forward to calculate contact stresses on their teeth. Contact failure in gears is currently predicted by comparing the calculated Hertz con-tact stress to experimentally determined allowable values for the given material. The method of calculating gear con-tact stress by Hertzian equation originally derived for con-tact between two cylinders. Analytically these contact stresses are calculated for different module, and these re sults are compared with the results obtained in modeling analysis in ANSYS.

## III. GEAR GEOMETRY AND MATERIAL PROPERTIES

The Lathe machine gear pair considered is a Grey Cast Iron Involute profile Spur gear. The geometry dimensions are defined in Table no. 1 and material properties are given in Table no. 2.

**Table 1 Dimensions of Gear**

| Dimension | Unit | Values |
|---|---|---|
| No. of teeth | - | 20 |
| PCD | mm | 127 |
| Pressure Angle | Deg | 20 |
| Addendum radius | mm | 69.85 |
| Dedendum radius | mm | 55.88 |
| Face Width | mm | 25.4 |

**Table 2 Material Properties**

| Material Property | Unit | Grey Cast Iron |
|---|---|---|
| Density | Kg/m3 | 7200 |
| Young's modulus | Pa | 1.1E+11 |
| Poisson's Ratio | - | 0.28 |
| Bulk Modulus | Pa | 8.33E+10 |
| Shear Modulus | Pa | 4.29E+10 |

## IV. ANALYTICAL CONTACT STRESS CALCULATION (HERTZ THEORY)

The transfer of power between gears takes place at the contact between the acting teeth. The stresses at the contact point are computed by means of the theory of Hertz. The theory provides mathematical expressions of stresses and deformations of curved bodies in contact. Fig. 1 shows a model applied to the gear-two parallel cylinders in contact.[2]

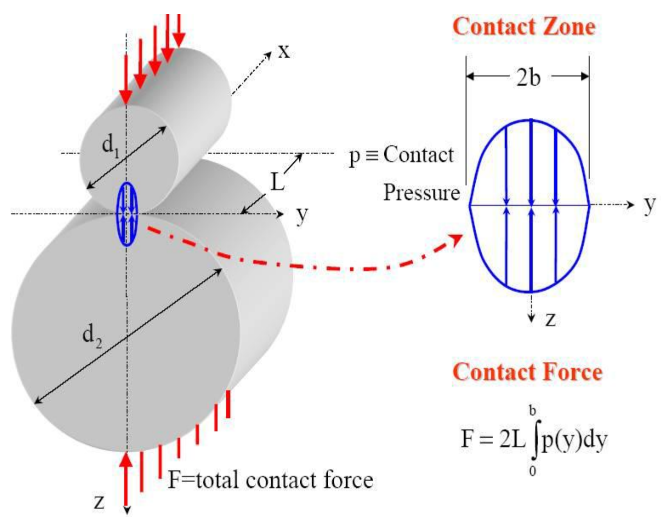


**Figure 1 Hertz Theory [3]**

So, the hertz equation for contact stresses in the gear teeth is,

$$\sigma_c = \sqrt{\frac{F(\frac{1}{R_1}+\frac{1}{R_2})}{L*\pi*[\frac{(1-\upsilon_1^2)}{E_1}+\frac{(1-\upsilon_2^2)}{E_2}\}}}$$

Where,

F = Contact Force  L = Face width
$R_1$ = Radius of Pinion  $R_2$ = Radius of Gear
$\upsilon_1$ & $\upsilon_2$ = Poisson's Ratios of Pinion & Gear
$E_1$ & $E_2$ = Young's Modulus of pinion & Gear

The Torque provided by the motor to the gearbox is 70028.17 N-m so the contact stress produced is 2334.27 N for the 300 rpm. So, the equation gives the result as,

$$\sigma_c = \sqrt{\frac{2334.27\left(\frac{1}{54}+\frac{1}{60}\right)}{12*\pi*\left[\frac{(1-0.35^2)}{118000}+\frac{(1-0.35^2)}{118000}\right\}}}$$

$$\sigma_c = 382.72\,Mpa$$

## V. FINITE ELEMENT ANALYSIS FOR CONTACT STRESSES

The Gear Pair with module 2 is modeled in Creo2.0 and later imported in Ansys14.5 for the static analysis. The fixed support is given at the inner rim of the pinion whereas the frictionless support is given at rim of the gear and the torque is applied at the gear. The fig. 2 shows the boundary conditions and fig. 3 shows the meshed model where different size of mesh is used at different locations to get the most accurate results.

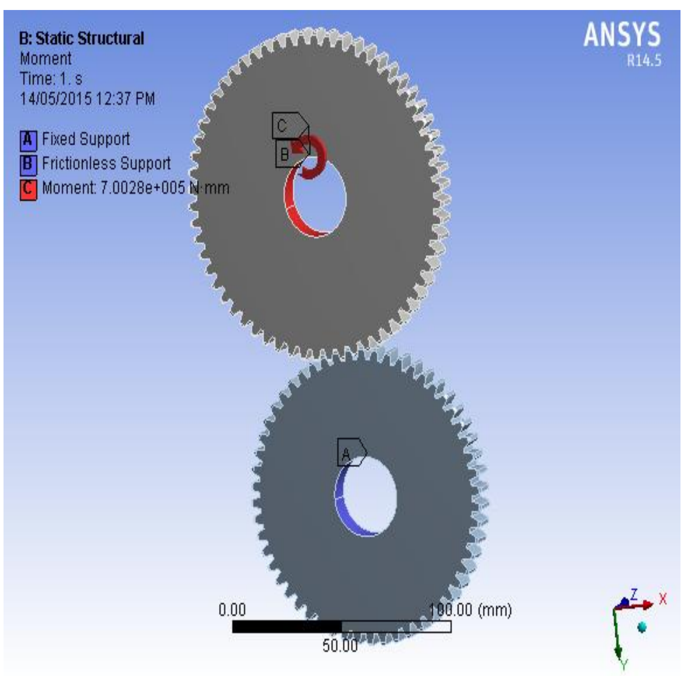


**Figure 2 Boundary Conditions**

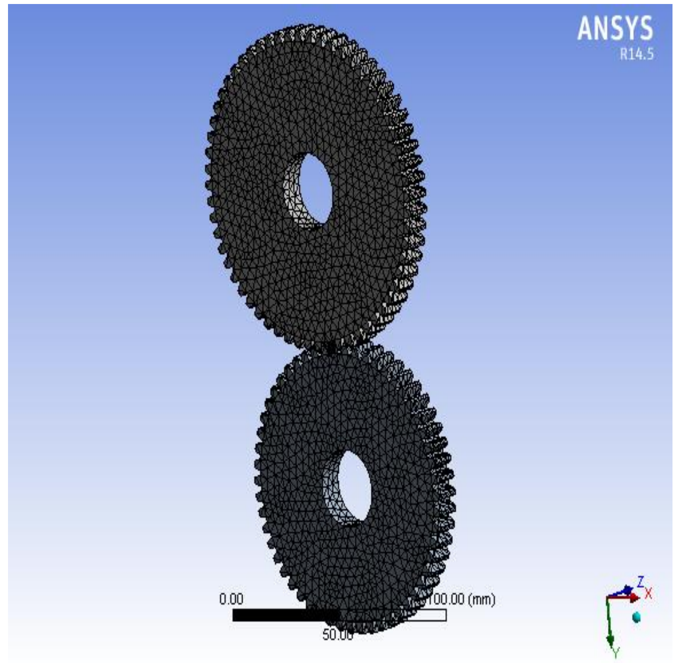


**Figure 3 Meshed Model**

The gear pair model with module 2 and the given boundary conditions are analyzed for contact stresses under static loading conditions in the Mechanical Modeler of Ansys14.5. The results of gear pairs with module 2 and 3 are shown in fig.4 & fig.5 respectively.

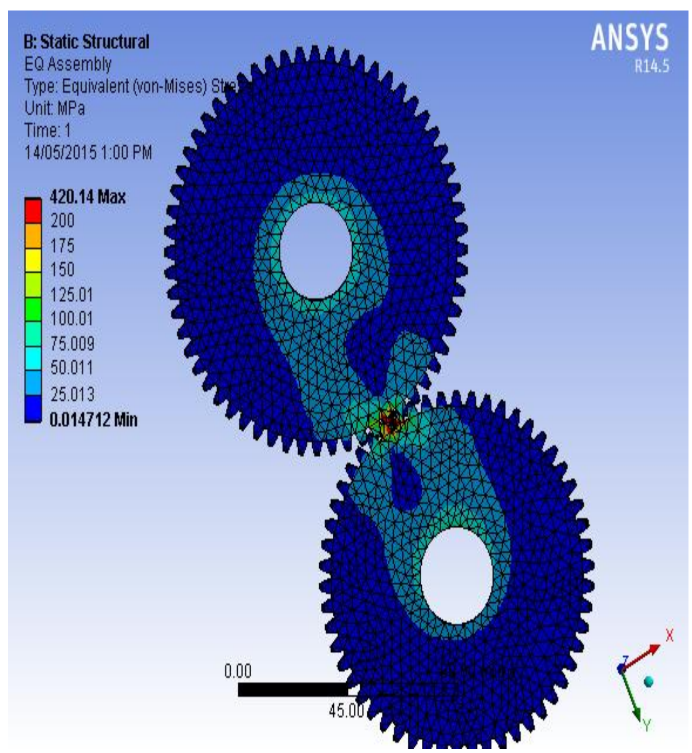


**Figure 4 Stresses in Gear with Module 2**

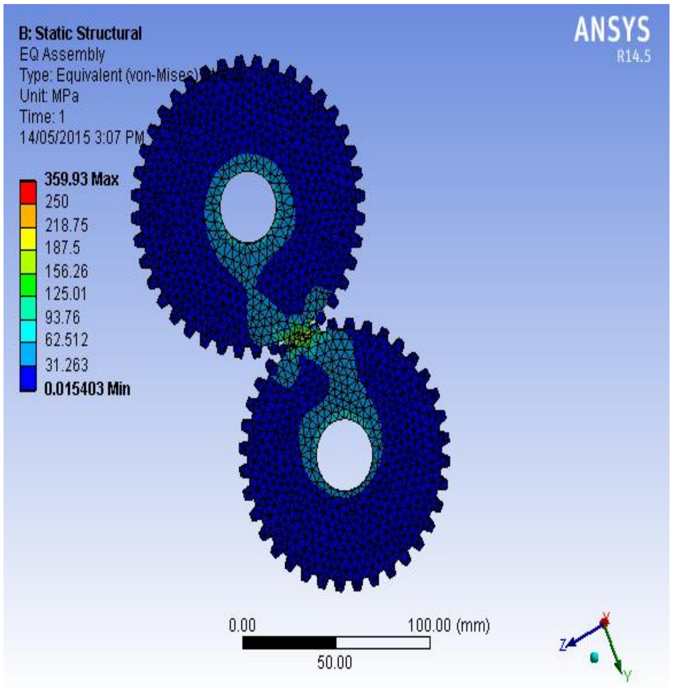


**Figure 5 Stresses in Gear with Module 3**

## VI. RESULTS & DISCUSSIONS

The results of Hertz contact stresses and finite element analysis for modules ranging from 2 to 5 are compared in table no. 3.

**Table 3 Comparison of Hertz theory and FEA**

| Gear Modules | Hertz Contact Stresses (mpa) | Contact Stresses by FEA (mpa) | Percentag e difference (%) |
|---|---|---|---|
| 2 | 382.720 | 420.14 | 8.90 |
| 3 | 378.578 | 359.93 | 4.92 |
| 4 | 374.560 | 356.63 | 4.78 |
| 5 | 370.667 | 348.73 | 5.91 |

The above table shows that the contact stresses is directly proportional to the module of the gear, as the module increases the contact stresses decreases. Both the Hertz theory as well as the finite element analysis shows the same pattern of results, although the nature of hertz contact stresses is linear but the finite element result shows non-linear nature which is probably due to the size and type of discretization process (meshing).

## VII. CONCLUSION

The Contact stresses developed under a spur gear pair of lathe machine under static loading is calculated in this study analytically as well as using finite element method. Both the Hertz theory of contact stresses as well as FEM method of contact stresses gives almost identical results with a mere percentage difference. The Hertz theory is a quick process to calculate the contact problem within its assumptions, whereas the finite element method has a detailed approach with high sensitivity towards its boundary conditions and discretization methods. Even there is diversity in approach but both the methods shows similar outline in results.

## REFERENCES

[1] X. Feng, “Analysis of Field of Stress and Displacement in Process of Meshing Gears” *International Journal of Digital Content Technology and its Applications, Volume 5, Number 6*, June 2011, 345-357.

[2] Ali Raad Hassan, “Contact Stress Analysis of Spur Gear Teeth Pair”, *World Academy of Science, Engineering and Technology*, Vol. 3, 2009, pp. 597-602.

[3]M. Raja, P. Phani, "Contact pressure analysis of spur gear using FEA", *International Journal of Advanced Engineering Applications, Vol.7, Iss.3*, 2014 pp.27-41.

[4]Bharat Gupta, Abhishek Choubey, Gautam V. Varde , Contact Stress Analysis Of Spur Gear,Vol.1

[5]Vivek Karaveer, T. Mogrekar, “Modeling and Finite Element Analysis of Spur Gear”, *International Journal of Current Engineering and Technology*, Vol3., No.5, December 2013.

[6]V. Rajprabhakaran, R. Ashokraj, “ Spur Gear Tooth Stress Analysis And Stress Reduction”, *IOSR Journal of Mechanical and Civil Engineering*, pp. 38-48.

[7]S. Tiwari, U. Joshi, “Stress Analysis of Mating Involute Spur Gear Teeth”, *International Journal of Engineering Research & Technology (IJERT) Vol. 1 Issue 9*, November- 2012.

[8]S. Shinde, A. Nikam “ Static Analysis of Spur Gear Using Finite Element Analysis”, *IOSR Journal of Mechanical and Civil Engineering*, PP: 26-31.

[9]Sunil Kumar, K.K. Mishra, Jatinder Madan, Stress Analysis Of Spur Gear Using FEM method , *National Conference of Advancementsand Futuristic Trends in Mechanical and Material Engineering* , Feb 19.20.2010.

[10]Sushil Kumar Tiwari, Upendra Kumar Joshi, Stress Analysis of moving involute spur gear teeth *ISSN:2278-0181*.

[11]Kailash C. Bhosale, Analysis of bending strength of the helical gear by FEM, *ISSN 2222-1727(Paper) ISSN 2222-2871(Online) Vol 2,No 4, 2011*.