Frequency Response of 10 Degrees of Freedom
Full-Car Model For Ride Comfort

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ABSTRACT
A mathematical model for a full vehicle dynamic system of 10 degrees of freedom is derived using Lagrange’s equation. The model is used to simulate the vibration response of the vehicle according to a road having sinusoidal disturbance for exciting frequencies up to 18 Hz. All the vehicle natural frequencies are evaluated. The frequency response of nine of the full-car model variables is simulated using MATLAB. The vibration level of the driver and passengers are computed in m²/s (root mean square) and compared with the recommended exposure time according to ISO 2631-1 standard. The study showed that the passengers are safe (ride comfort fulfilled) for exposure time limits up to 24 hours. The driver bounce failed to pass the 24 hour exposure time for car speeds between 25 and 40 km/h.

Keywords- Vehicle dynamics , 10 DOF full-car model , frequency response , ISO 2631, passengers and driver bounce.

I. INTRODUCTION
Frequency response is a well known technique used to investigate the dynamics of engineering physical systems including vehicles. It is equally applied to different types of vehicle models including quarter, half and full-car models. In this study full 10 DOF car model is used to investigate the dynamics of the vehicle using the frequency response technique.

McCann (2000) showed that using the yaw rate feedback improves vehicle stability. He presented the input-output frequency response of the driver-vehicle system for up to 100 rad/s frequency [1]. Lombaert and Degrande (2001) used a 4 DOF vehicle model to calculate the axle loads from the longitudinal road profile. They calculated the free field soil response using a transfer function between the road and receiver. They obtained the frequency response for a Volvo truck for frequencies up to 100 Hz [2]. Sammier, Sename and Dugard (2002) used H∞ and Skyhook control to improve comfort applied to a single-wheel suspension car model. They presented the frequency response of the vehicle for open and closed loop for frequencies up to 10 kHz [3]. Pick and Cole (2003) studied the role of the steering torque feedback in the vehicle driver system. They presented the frequency response of limb dynamics for up to 100 Hz frequency [4]. Freeman (2004) developed a nonlinear passive mount design to provide good energy absorption at higher amplitudes and higher frequencies and good vibration isolation at lower amplitudes and lower frequencies. He presented the frequency response of the vehicle system for frequencies up to 25 Hz [5].

Rix (2005) studied the identification of driver steering behaviour in closed-loop. He presented the vehicle model frequency response in the form of Bode plot for frequencies up to 100 Hz [6]. Melcer (2006) considered the quarter-car model and calculated its frequency response characteristics for frequencies up to 100 rad/s [7]. Zhang, Chen and Shangguan (2007) investigated the vehicle non-stationary process by using a fractional damping model. They acquired the vehicle non-stationary random response in the frequency domain using the FFT [8]. Zalm, Huisman, Steinbuch and Veldpaus (2008) discussed the modelling and model verification of a vehicular driveline and tuning of speed controllers using frequency domain techniques. The presented the frequency response of the vehicle for frequencies up to 100 Hz [9]. Detweiler (2009) presented an identification method to obtain better estimate of the inaccessible parameters of the vehicle model using measured terrain excitations. He demonstrated the frequency response function of a 7 DOF model for frequencies up to 100 Hz [10]. Zhang, Cheung and Cheng (2010) analysed the frequency response of a quarter-car model using Bode plot to investigate the influence of suspension parameters on system
characteristics. They covered frequencies up to 1000 rad/s [11].

Barbosa (2011) used a half-car dynamic model to develop a spectral method to obtain the frequency response of his model subjected to a measured pavement roughness in the frequency domain. He covered a frequency range from 0.1 to 30 Hz [12]. Renuke (2012) investigated the vibration characteristics of a car chassis including the natural frequencies and mode shapes. He used the finite elements method for this purpose. He showed that the chassis natural frequencies (42.4 – 93.7 Hz) lie within the road excitation frequency range [13]. Raju and Venkatachalam (2013) derived the equations of a 7 DOF full-car model using a semi-independent suspension and studied the dynamic behaviour of the system through frequency response. They covered frequencies up to 32 Hz since the natural frequencies of the system were in the range 1.79-18.88 Hz [14]. Mahmood and Khan (2014) investigated the responses of the quarter-car and half-car models. They studied the time and frequency responses of the sprung and unsprung masses. They optimized the damping of the suspension for a given set of fixed parameters [15]. Kong and Kong (2015) conducted a parametric study on measurement numbers, road roughness and vehicle speed to analyze the effectiveness of using those parameters for damage detection using the transmissibility of the vehicle-bridge coupled system. They extracted the natural frequencies and mode shapes for the damage detections [16].

II. THE MODELLED VEHICLE

A simplified physical model of the vehicle is shown in Fig.1

![Fig.1 The full-car model.](image)

There are 10 degrees of freedom for the vehicle system of Fig.1 As follows:

1) Vehicles chassis vertical motion in Z-axis.
2) Vehicles chassis pitching (about Y-axis) (Ψ).
3) Vehicles chassis rolling (about X-axis) (Ø).
4) Vehicles chassis lateral motion (y).
5) Driver bounce \( z_5 \) (vertical motion in Z-axis).
6) Passengers bounce \( z_6 \) (vertical motion in Z-axis).
7) Front right wheel bounce \( z_1 \).
8) Front left wheel bounce \( z_2 \).
9) Rear left wheel bounce \( z_3 \).
10) Rear right wheel bounce \( z_4 \).

The model is exited at wheels by motions \( z_{11}, z_{22}, z_{33} \) and \( z_{44} \) which results from the travelling of vehicle over the sinusoidal hump.

III. VEHICLE SIMULATION PARAMETERS

The parameters of the vehicle used in this work are given in Table 1 [17].

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M_1 ) (kg)</td>
<td>1100</td>
<td>( c_3, c_4 ) (Ns/m)</td>
<td>2500</td>
</tr>
<tr>
<td>( m_1, m_2 ) (kg)</td>
<td>25</td>
<td>( c_5 ) (Ns/m)</td>
<td>150</td>
</tr>
<tr>
<td>( m_3, m_4 ) (kg)</td>
<td>45</td>
<td>( c_6 ) (Ns/m)</td>
<td>300</td>
</tr>
<tr>
<td>( m_5 ) (kg)</td>
<td>90</td>
<td>( k_{11}, k_{22} ) (N/m)</td>
<td>250000</td>
</tr>
<tr>
<td>( m_6 ) (kg)</td>
<td>180</td>
<td>( k_{33}, k_{44} )</td>
<td>250000</td>
</tr>
<tr>
<td>( I_x ) (kg.m^2)</td>
<td>550</td>
<td>( k_{11}, k_{22} )</td>
<td>5250</td>
</tr>
<tr>
<td>( I_y ) (kg.m^2)</td>
<td>1848</td>
<td>( k_{33}, k_{44} )</td>
<td>5250</td>
</tr>
<tr>
<td>( k_{11}, k_{22} ) (N/m)</td>
<td>1500</td>
<td>a (mm)</td>
<td>1200</td>
</tr>
<tr>
<td>( k_{33}, k_{44} ) (N/m)</td>
<td>1700</td>
<td>b (mm)</td>
<td>1400</td>
</tr>
<tr>
<td>( k_5 ) (N/m)</td>
<td>15000</td>
<td>L_1 (mm)</td>
<td>500</td>
</tr>
<tr>
<td>( k_6 ) (N/m)</td>
<td>30000</td>
<td>e (mm)</td>
<td>300</td>
</tr>
<tr>
<td>( c_1, c_2 ) (Ns/m)</td>
<td>2500</td>
<td>d (mm)</td>
<td>250</td>
</tr>
</tbody>
</table>

The road disturbances are sinusoidal with the following values:
- Wave length (\( L_h \)): 4.5 meter.
- Wave peak amplitude (H): 50 mm
IV. DYNAMIC SYSTEM SIMULATION

4.1 System Natural Frequencies
A complete mathematical model of the vehicle with 10 DOF is derived using Lagrange’s equation and used in previous time analysis work [18]. The MATLAB software program is used to solve the differential equations of motion by using the central difference method as a numerical integration technique [19]. The mathematical model is constructed by applying the Lagrange’s equation to the system and writing the equations in matrix form. MATLAB is used to assign the system. The simulation analysis results of the system are given in Table 2.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Undamped Natural Frequency (rad/s)</th>
<th>Undamped Natural Frequency (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>4.3693</td>
<td>0.695</td>
</tr>
<tr>
<td>(2)</td>
<td>5.1761</td>
<td>0.823</td>
</tr>
<tr>
<td>(3)</td>
<td>6.4079</td>
<td>1.019</td>
</tr>
<tr>
<td>(4)</td>
<td>6.7125</td>
<td>1.068</td>
</tr>
<tr>
<td>(5)</td>
<td>13.4955</td>
<td>2.147</td>
</tr>
<tr>
<td>(6)</td>
<td>15.8647</td>
<td>2.524</td>
</tr>
<tr>
<td>(7)</td>
<td>77.0346</td>
<td>12.26</td>
</tr>
<tr>
<td>(8)</td>
<td>77.0564</td>
<td>12.26</td>
</tr>
<tr>
<td>(9)</td>
<td>102.96</td>
<td>16.38</td>
</tr>
<tr>
<td>(10)</td>
<td>102.97</td>
<td>16.38</td>
</tr>
</tbody>
</table>

4.2 The Frequency Response
The frequency response of the 10 DOF full-car model to a sinusoidal input at the ground having exciting frequency up to 18 Hz is shown in Fig.2 to 10 for the different elements of the full-car model.
V. STATISTICAL PARAMETERS

- The frequency response of the model elements changes with exciting frequency.
- Within the frequency range from 0 to 28 Hz, the dynamic motion changes between minimum and maximum values.
- Using the MATLAB commands 'min' and 'max', those values are defined for all the model elements [20].
- The results are presented in Table 3.
TABLE 3
Minimum and Maximum Frequency Response Data

<table>
<thead>
<tr>
<th>Motion</th>
<th>Description</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>z (mm)</td>
<td>Bounce of chassis z-axis</td>
<td>1.257</td>
<td>24.38</td>
</tr>
<tr>
<td>y (mm)</td>
<td>chassis lateral motion</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Θ (rad)</td>
<td>chassis rolling (about X-axis)</td>
<td>1.4e-08</td>
<td>0.0067</td>
</tr>
<tr>
<td>z1 (mm)</td>
<td>Bounce of right front wheel in z-axis</td>
<td>40.78</td>
<td>58.5</td>
</tr>
<tr>
<td>z2 (mm)</td>
<td>Bounce of left front wheel in z-axis</td>
<td>40.78</td>
<td>58.5</td>
</tr>
<tr>
<td>z3 (mm)</td>
<td>Bounce of right rear wheel in z-axis</td>
<td>11.02</td>
<td>30.29</td>
</tr>
<tr>
<td>z4 (mm)</td>
<td>Bounce of left rear wheel in z-axis</td>
<td>11.03</td>
<td>30.29</td>
</tr>
<tr>
<td>z5 (mm)</td>
<td>Bounce of driver in z-axis</td>
<td>0.030</td>
<td>116.8</td>
</tr>
<tr>
<td>z6 (mm)</td>
<td>Bounce of passengers in z-axis</td>
<td>0.017</td>
<td>53.31</td>
</tr>
</tbody>
</table>

VI. WHOLE BODY VIBRATIONS

The whole body vibration of human body is transmitted to the body through the supporting surfaces such as the feet, buttocks or back. There are various sources of whole body vibration such as standing on a vibrating platform, floor surface, driving, and construction, manufacturing, and transportation vehicles [21].

The effects of whole body vibration exposure for human may be bio-dynamics, psychological, physiological or pathological [22]. The level of effects depends on the frequency, acceleration and the time of exposure (long-term or short term exposure).

VII. THE STANDARD ISO 2631

ISO 2631 was first published in 1974, with the purpose of giving numerical values for limits of exposure for vibrations transmitted from solid surfaces to the human body in the frequency range 1 to 80 Hz. The standard was republished in 1978, 1982, 1985 and 1997, with major revisions along the way. The current version of ISO 2631, entitled “Mechanical vibration and shock – Evaluation of human exposure to whole-body vibration” consists of four parts (labeled 2631-1, 2631-2, 2631-4 and 2631-5). The part that is relevant to the current paper is 2631-1, which is called “General Requirements.” ISO 2631-1:1997 provides a more quantitative guide on the effects of vibration on health and comfort [23,24].

The vibration acceleration limits based on the period of exposure from 1 min until 24 h of ISO 2631-1 are shown in Fig.11 [24].

![Fig.11 The vibration acceleration limits (ISO 2631-1)[24].](image)

VIII. ISO 2631 LIMITS AND DRIVER AND PASSENGERS VIBRATIONS

The comparison between the passengers and driver bounces acceleration in root mean square form with respect to the ISO 2631 standards is shown in Figs.12 and 13. The passengers vibration is accepted for all the exposure limits except for the 24 hours limit where the condition is marginal at an exciting frequency of 2 Hz (30 km/h car speed).

The driver vibration are accepted for all the exposure limits up to 16 hours exposure. The 24 limit exposure is violated in the frequency range 1.66 to 2.66 Hz (between 25 and 40 km/h car speeds).
The effects of whole body vibration exposure for human may be bio-dynamics, psychological, physiological or pathological [22]. The level of effects depends on the frequency, acceleration and the time of exposure (long-term or short term exposure).

IX. CONCLUSION

- A 10 DOF full-car model was used in this work to investigate the car dynamics.
- The frequency response of the 10 DOF full-car model was investigated.
- Ten natural frequencies of the full-car model were calculated.
- The system natural frequencies were in the range: 0.695 to 16.38 Hz.
- The frequency response of 9 of the full-car variables was presented for exciting frequencies up to 18 Hz.
- The exposure of the passengers on the rear seat and the driver according to ISO 2631-1 was investigated.
- It has been shown that the passengers were safe for all the exposure limits up to 24 hours and for the frequency range up to 18 Hz.
- It has been shown that the driver bounce violated the 24 hours limit at car speeds in the range: 25 to 40 km/h (1.66 to 2.66 Hz).

X. REFERENCES


Fig.12 Passengers bounce acceleration and ISO2631 limits.

Fig.13 Driver bounce acceleration and ISO2631 limits.
Polytechnic Institute, State University, April 2009.


**BIOGRAPHY**

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- Emeritus Professor of System Dynamics and Automatic Control.
- Has got his Ph.D. in 1979 from Bradford University, UK under the supervision of Late Prof. John Parnaby.
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