Effect of Samples on PSE Using Welch Method

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Abstract

Variation in data sequence length may change the performance of power spectrum estimation technique. In this paper PSE for Bartlett window technique for different samples has been perform using Welch method, which is a non-parametric method or some times called averaging modified periodogram technique. This non-parametric PSE approach gives good resolution result if data length or number of samples has been chosen in a precise way. This paper gives the matlab result for Bartlett window technique using Welch method for different samples under consideration and shows that increased finite number of samples gives better resoluted result compared to less number of samples.

Keywords- *Bartlett window, PSD, PSE, Quality factor, Welch method.*

I. INTRODUCTION

In this paper we wish to show the effect of sample variation on PSD with the help of Welch method for Bartlett window. The basic problem that occurs is the estimation of power spectral density of a signal from the observation of the signal over finite time interval. The finite record length of the data sequence is a major limitation on the quality of the power spectrum estimation [1].The non-parametric Welch in which the power of any point is guesstimation at different frequencies [2].When dealing with the signals that are statistically stationary, the longer data record, the better the estimate that can be extracted from the data. On the other hand if the signal statistics are non-stationary we can not select an arbitrarily long data record to estimate the spectrum. Here in this paper we plot the PSE graph for the sum of two sinusoidal waveform with noise content and its variation over distinct samples. Bartlett window has admirable resolution characteristics for sinusoids of comparable strength Bartlett method provide a method to reduce the variance of periodogram in exchange for a reduction of resolution in compared to standard periodogram [3].Welch method is a modified version of periodogram or Bartlett in which portion of the series contributing to each periodogram are used to overlap. Here all estimations come under 50% overlapping of each segment of window which is the conventional way to use Welch method for better resolution.

II. POWER SPECTRUM ESTIMATION

Power spectrum estimation concerned with the spectral characteristics of signal characterized as random process many of the phenomena that occur in nature are best characterized statistically in terms of averages. For example, meterological phenomena such as fluctuations in air temperature and pressure are best characterized statistical random processes. Power spectrum estimation methods have a relatively long history. For a historical perspective , the reader is referred to the paper by Robinson(1982) and the book by Marple (1987).the classical power spectrum estimation methods based on the periodogram, originally introduced by Schuster (1898) and by Yule(1927), who originated the modern model-based or parametric methods. These methods were subsequently developed and applied by Walker(1931), Bartlett(1948), Parzen(1957), Blackman and Tukey(1958), Burg(1967) and others. One of the problems that we encounter with classical power estimation methods based on a finite-length data record is the distortion of the spectrum that we are attempting to estimate. This problem occuers in both the computation of the spectrum for a deterministic signal and the estimation of the power spectrum of a random signal. Since it is easier to observe the effect of the finite length of the data record on a deterministic signal.

a. Welch method

The following two modifications were made by Welch in 1967 in the averaging periodogram.

(1) The subsequence of x(n) are applied to overlap.

(2) A data window w(n) is applied to each subsequence in computing the periodograms.

If we are talking about Welch method, the overlapping subsequence are represented by

Xi(n)=x(n+iD), n=0,1,-----M-1

I=0,1,-----L-1 (1)

iD is nothing but starting point of the sequence.

Observe that if D=M, segments do not overlap and the L of data segment is identical to number K in Bartlett method.

If D=m/2 there is 50% overlapping between successive data segments and L=2k segments are obtained. Alternatively we can form K data segments each of length 2M.

Again in second modification made by Welch method to Bartlett method is to window tha data segments prior to computing the periodogram , the result is a "modified" periodogram.

$$\breve{P}^{i}_{xx}(f) = \frac{1}{MU}\left|\sum_{n=0}^{M-1} x_i(n)w(n)e^{-j2\pi fn}\right|^2 \quad i = 0,1,\ldots..L-1 \tag{2}$$

Where U is a normalized factor for the power in the window function and is selected as-

$$U = \frac{1}{M}\sum_{n=0}^{M-1} w^2(n) \tag{3}$$

The Welch power spectrum estimate is the average of these modified periodogram, that is-

$$P^{W}_{XX} = \frac{1}{L}\sum_{i=0}^{L-1} \breve{P}^{i}_{XX}(f) \tag{4}$$

The mean value of the Welch estimate is-

$$E[P^{W}_{XX}(f)] = \frac{1}{L}\sum_{i=0}^{L-1} E\breve{P}^{i}_{XX}(f) \tag{5}$$

in case of 50% overlapping between successive data segments (L=2K), the variance of the Welch power spectrum estimate with the triangular window also derived in the paper by Welch is-

$$\mathrm{var}[p^{W}_{xx}(f)] = 9/82\ \Gamma^{2}_{XX(f)} \tag{6}$$

the quality factor of Welch method may be written as-

L = N/M for no overlapping

8L/9 = 16N/9M for 50% overlapping (7)

The spectral width of triangular window is at 3Db point is given as-

$$\Delta f = 1.28/M \tag{8}$$

The quality factor expressed in terms og N and Δf

$$Q_w = \begin{cases} 0.78N\Delta f \\ 1.39N\Delta f \end{cases} \tag{9}$$

Variance of Welch method is given as----

$$\mathrm{var}\left[P^{w}_{xx}(f)\right] = \frac{1}{L^2}\sum_{i=0}^{L-1}\sum_{j=0}^{L-1} E\left[\overline{P}^{(i)}_{xx}(f)\overline{P}^{(j)}_{xx}(f)\right] - \{E[P^{w}_{xx}(f)]\}^2 \tag{10}$$

Properties of Welch method-

$$\hat{r}_W(e^{j\omega}) = \frac{1}{KLU}\sum_{i=0}^{k-1}\left|\sum_{n=0}^{L-1} x(n+iD)e^{-jn\omega}\right|^2$$

$$\hat{P}_W(e^{j\omega}) = \frac{1}{L}\sum_{i=0}^{k-1}\hat{r}^{(i)}_M(e^{j\omega}) \quad , \quad U = \frac{1}{L}\sum_{n=0}^{L-1}|w(n)|^2$$

Bias

$$E\{\hat{P}_B(e^{j\omega})\} = \frac{1}{2\pi LU} P_x(e^{j\omega}) * \left|W(e^{j\omega})\right|^2$$

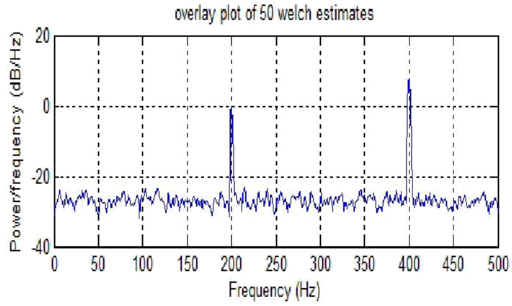
TABLE I. Non-parametric methods of power spectrum estimation

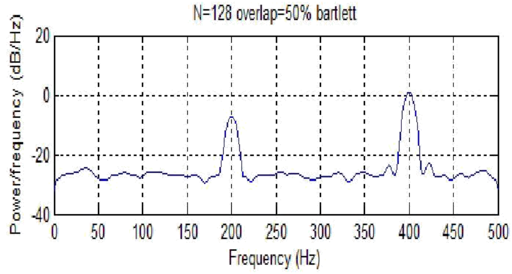
	Quality factor Q	Resolution $\Delta\omega$	Figure of merit M
Periodogram	1	$0.89\frac{2\pi}{N}$	$0.89\frac{2\pi}{N}$
Bartlett	1.11NΔf	$0.89k\frac{2\pi}{N}$	$0.89\frac{2\pi}{N}$
Welch	1.39NΔf	$1.28\frac{2\pi}{L}$	$0.72\frac{2\pi}{N}$
Blackman Tukey	2.34NΔf	$0.64\frac{2\pi}{M}$	$0.43\frac{2\pi}{N}$

III. RESULTS

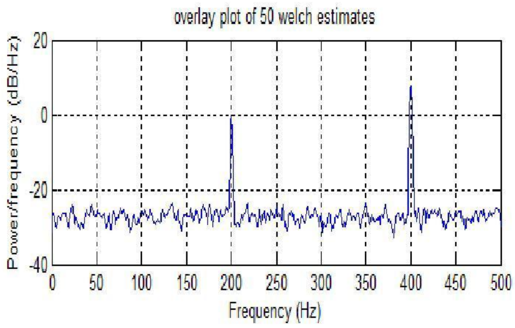
In this paper we are trying to show the effect of variation of number of samples on resolution of Welch method.Here we are adding two sinusoidal waveforms plus noise under sampling frequency fs = 1000 and 50% overlapping.

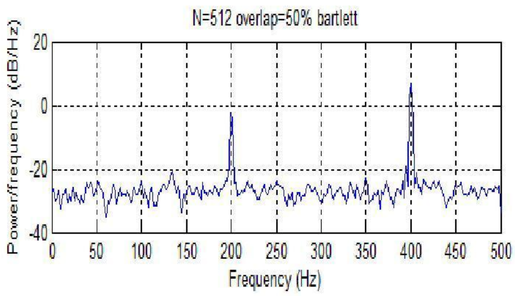
Again here we are taking Bartlett window as a reference and start with N = 128 and fs = 1000, we get result as-



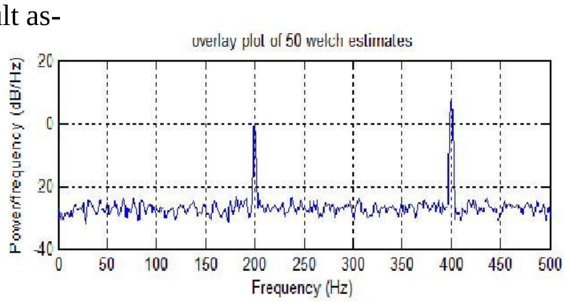


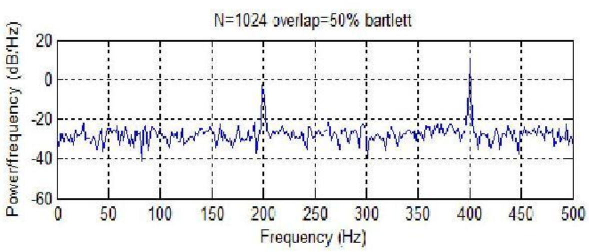
Again start with Bartlett window as a reference, this time we take N = 512, fs = 1000, we get result as-





Again here we start with Bartlett window, but this time we take N = 1024 and fs = 1000, we get the result as-





IV. CONCLUSION

On the basis of the result that we have shown N=128,N=512 and N=1024, we can make the conclusion based on the observation that N=1024, gives the better resolution and variance compared to N=128 and N=512. This shows that Welch method is suitable if we increase Number of samples or can say that this method gives better Result if we increase samples. Comparison have done with Bartlett window which alsogives the result that it have clear peak as we increase the number of samples as shown in graph.

Thus we can conclude the whole result that Welch method gives psd of a signal with reducing the effect of noise &work in a better way if we increase the number of samples.

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