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About instability of current of a layer of a liquid upper a sandy bottom

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1. Introduction

Huge interest is caused with researches of wave formation in currents above a sandy bottom. Wave structures in such currents on the mechanism of formation and the characteristics divide in two groups. The wave movements connected to presence of a free surface of a stream concern to the first group; they exist and at current of a liquid above not deformable bottom. These waves are formed at enough big numbers $Fr > Fr_c$. At small speeds of a liquid at which Froud's number essentially less than critical value Fr_c, but it is enough for the beginning movement of ground particles in a narrow benthonic layer or in a suspension, occurrence of wave structures of the second type is possible. Waves which size is comparable or exceeds depth of a stream, refer to as dunes; ground structures of the smaller sizes frequently define as a ground ripples. It is necessary to note, that the mechanism of instability, the leader to formation of superficial or ground waves, also causes deformation of other stream border.

Interest to such currents is connected with designing and operation of hydraulic engineering constructions. In currents above a sandy bottom a plenty of works (1-6 where it is possible to find the further references to work of other authors) is devoted to research of wave formation

2. The Mathematical Model

Used mathematical models differ by ways of the description of movement of a liquid and ground deposits. Formation of waves on the bottom surface of a stream has been received with use of the model of potential current of an ideal liquid within the framework of systems of the hydraulic equations or application half empirical models of turbulence. In potential and hydraulic models instability of the ground surface was defined by introduction of artificial shift between the charge of ground particles and local characteristics of a stream of a liquid. In the given work is offered the hydraulic model which includes both mechanisms of instability and does not contain artificial shift of phases.

The equation of hydraulics is received from equation Navier-Stokes where, instead of a viscous member, is entered the hydraulic resistance $a\frac{|\vec{3}|\vec{3}}{2h}$. Depending on a kind of hydraulic resistance critical number Froude Fr_c will be miscellaneous; in our case $Fr_c = 2$. If consider hydraulic resistance as $a\frac{|\vec{3}|\vec{3}}{2h^2}$, or $a\frac{|\vec{3}|\vec{3}}{2h^{\frac{3}{2}}}$ that number Froude accordingly will be miscellaneous. In the first case $Fr_c = 1$. Transform to dimensionless variables depends on the flow scales.

Used approaches to theoretical studying and numerical modeling of processes of deformation of a sandy bottom in the majority it is limited only to research of movement bedload of the deposits moving in a bottom and in immediate proximity from it. Thus are used as empirical, and half impirical dependences for the charge bedload deposits.

The weighed particles have significant volume of deposits (in particular in fine fractions). Deposits can be lifted from a bottom and as a suspension are transferred on distance where they will be gradually besieged. Therefore, at research of rearrangement of a sandy bottom, alongside with bedload deposits it is necessary to take into account movements of the weighed deposits, and also process of an exchange by deposits between a bottom and the basic thickness of a stream. Significant difficulties of research of transport of the weighed deposits are connected to development of models of vertical carry of particles near to a bottom which were to some extent adequate to the observable physical phenomenon.

For more exact account of dynamics of interaction of a non-uniform stream with a deformable bottom the two-layer mathematical model of movement of deposits is considered. The stream divides into a benthonic layer and a turbulent nucleus. Move of particles to a turbulent nucleus is described by the equation of turbulent diffusion in approach of fine water. Bedload transport change of marks of a bottom is described by the equation of indissolubility with attraction of the empirical formula for the charge of deposits. Between a benthonic layer and a turbulent nucleus the exchange of particles which pays off on the basis of the circuit of a turbulent exchange of V.M.Makkaveeva-A.V.Karausheva [5] is carried out.

3. Statement of a problem

We shall consider the current of an incompressible viscous liquid on an inclined plane by gravity. We shall choose system of coordinates X,Y,Z with an axis X directed on the current. The axis Z is focused aside liquids. We shall designate a surface of a bottom through $z_b(x,y,t)$. The corner of an inclination of a plane (X,Y) to horizon will be ¯. A free surface we shall set as $z=h(x,y,t)$. We shall assume, that in area $z_b + ¢ \le z \le h$ there are weighed deposits which movements are carried out by jumps and rollings. And in area $z_b + ¢ \ge z \ge z_b$ (a benthonic layer $¢ << h$) are available bedload deposits which are transferred by current as a heavy passive impurity. We shall consider, that between a turbulent nucleus and a benthonic layer there is an exchange of particles weighed and deposits thro bedload dough the area of a surface $z = z_b + ¢$. The charge of the weighed deposits are $s \cdot (h - ¢) \approx sh$, where s - concentration of the weighed deposits.

Bedload transport deposits we shall describe as a vector of the specific volumetric charge of particles $\vec{q} = (q_x, q_y)$. The volumetric charge of particles through the areas of a surface $z = z_b + ¢$ is described through q_0.

Let's consider the equation of hydraulics. Dimensionless variables are defined according to equality

$$X = H_c x \ , \quad Y = H_c y \ , \quad Z = H_c z \ , \quad T = \frac{H_c}{U_c} t \ ,$$

$$U(X,Y,T) = U_c u(x,y,t) \ , \quad V(X,Y,T) = U_c ³(x,y,t) \ ,$$

$$H_b(X,Y,T) = H_c z_b(x,y,t) \ , \quad H_s(X,Y,T) = H_c h_s(x,y,t)$$

where H_c, U_c are characteristic length and the speed determined below;

U, V are the components of speed corresponding to directions X and Y; H_b, H_s are levels of the ground and free surfaces; T is time. It is supposed, that components of speed U and V will be essence average on depth of a layer of value of their true sizes and Z is a component of speed and is equal to zero. For the entered sizes the system of the equations of hydraulics looks like

$$\begin{cases} \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + ³\frac{\partial u}{\partial y} = -\frac{\cos ¯}{Fr^2}\frac{\partial (z_b + h)}{\partial x} - ª\frac{u\sqrt{u^2 + ³^2}}{2h} + \frac{\sin ¯}{Fr_2}, \\ \frac{\partial ³}{\partial t} + u\frac{\partial ³}{\partial x} + ³\frac{\partial ³}{\partial y} = -\frac{\cos ¯}{Fr^2}\frac{\partial (z_b + h)}{\partial y} - ª\frac{³\sqrt{u^2 + ³^2}}{2h} , \\ \frac{\partial h}{\partial t} + \frac{\partial (uh)}{\partial x} + \frac{\partial (³ h)}{\partial y} = 0 , \end{cases}$$

(3.1)

$h = h_s - z_b$

where $Fr = \frac{U_c}{\sqrt{gH_c}}$ - number of Froude,

g - acceleration of a gravity; ª - factor of resistance, ¯ - a corner of an inclination of a plane (X,Y) to horizon. We note, that there are various dependences of the factor ª from parameters of a stream, for example, from h depth. The critical number of Froude is differant for the beginning formations of the waves connected to a free surface. Hier we assume $ª = const$. The third equation in (3.1) is the equation of indissolubility, averaged across on depth of a stream with the account boundary conditions on surfaces of a liquid. To system of the equations (3.1) it is necessary to add a relasion which describes deformation of the bottom surface $z_b(t,x,y)$ and which expresses the law of preservation of weight of ground deposits [5]

$$(1 - p_n)\frac{\partial z_b}{\partial t} + \frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} = -q_0$$

(3.2)

where q_x, q_y correspond to charges bedload deposits in directions x and y ,

q_0 is value characterizing the size describing mass transfer between bedload suspended particles, p_n is porosity of a material of a bottom. Carry of particles to a suspension and their soaring up or sedimentation are taken into account in the equation for concentration $s(t,x,y)$ of particles in a stream of a liquid

$$\frac{\partial(sh)}{\partial t}+\frac{\partial(suh)}{\partial x}+\frac{\partial(s{}^{3} h)}{\partial y}=q_0 \qquad (3.3)$$

Thus, the system of the equations (3.1) - (3.3) for unknown functions $u, {}^{3}, z_b, h, s$ is received. For its short circuiting necessary it is to use the ratio connecting sizes q_x, q_y and q_0 with parameters of a stream.

By virtue of that bedload particles deposits move in a thin layer at a bottom and are actuated, their charge reaches limiting value on distances about depth of a stream. It gives the basis in considered averaged through the depth of model to accept for calculation of the charge bedload particles one of formulas transport capacities. As a result of big amount of experimental data under various formulas has been established, that the best accuracy is given with formula Bagnold. Thus, as the charge
bedload deposits formula Bagnold in which influence of a local inclination of a bottom on the charge of deposits is in addition taken into account gets out

$$q_x=\frac{Au\sqrt{u^2+{}^{3\,2}}}{2}\left(\sqrt{u^2+{}^{3\,2}}-{}^{3}{}_n\right)-¥\,\frac{\partial z_b}{\partial x} \qquad (3.4)$$

$$q_y=\frac{A\,{}^{3}\sqrt{u^2+{}^{3\,2}}}{2}\left(\sqrt{u^2+{}^{3\,2}}-{}^{3}{}_n\right) \qquad (3.5)$$

where ${}^{3}{}_n=\frac{V_n}{U_c}$, V_n is value of not washing away speed, describing started movements of ground particles.

$$A=\lambda\frac{Fr}{\rho_p tg\,\alpha_b}\left(1-5.75\sqrt{\frac{\lambda}{2}}\lg\frac{0.37}{kd}-\frac{w_p}{\sqrt{u^2+\upsilon^2}}\right),$$

$$k=1.4\left(\frac{\sqrt{u^2+\upsilon^2}}{\upsilon_n}\right)^{0.6}$$

where ${}^{\circ}{}_p=\frac{{}^{\circ}{}_s}{{}^{\circ}}$. ${}^{\circ}{}_s$ and ${}^{\circ}$ are density of a material of ground particles and liquids accordingly; ${\sim}_b$ is angle of friction of a ground in a liquid, $d=\frac{D}{H_c}$, D is average diameter of particles. $w_p=\frac{W_p}{U_c}$, W_p is hydraulic size of particles, i.e. speed their sedimentation in a based liquid. Mass transfer between the ground and weighed particles q_0 is defined as follow

$$q_0=0.65\sqrt{\frac{\lambda}{2}}\left(0.00144\frac{Fr^2(u^2+\upsilon^2)}{\rho_p h}-\frac{Bs}{1-e^{-B}}\right),$$

$$B=1.8\frac{w_p}{\lambda\sqrt{u^2+\upsilon^2}} \qquad (3.6)$$

relation (3.4) - (3.6) define dependences of a kind $q_x(u,{}^{3},h), q_y(u,{}^{3},h)$ and $q_0(u,{}^{3},h,s)$

4. The solution of the problem

The problem (3.1) - (3.3) has the stationary solution

$$u_0=1,\ {}^{3}{}_0=0,\ (z_b)_0=0,\ h_0=1,\ s_0=\frac{0.00144Fr^2}{\text{a}\,{}^{\circ}{}_p B}\left(1-e^{-B}\right) \qquad (4.1)$$

corresponding to the current of a layer of constant thickness H_c above a deformable bottom; this solution exists by performance of equality

$$\frac{\sin\bar{}}{Fr^2}=\frac{\text{a}}{2} \qquad (4.2)$$

Which follows from the first equation (1) and which defines a choice of characteristic speed

$$U_c=\sqrt{\frac{2gH_c\sin\bar{}}{\text{a}}} \qquad (4.3)$$

For research of linear stability of current (4.1) solution of a non-stationary problem is represented as

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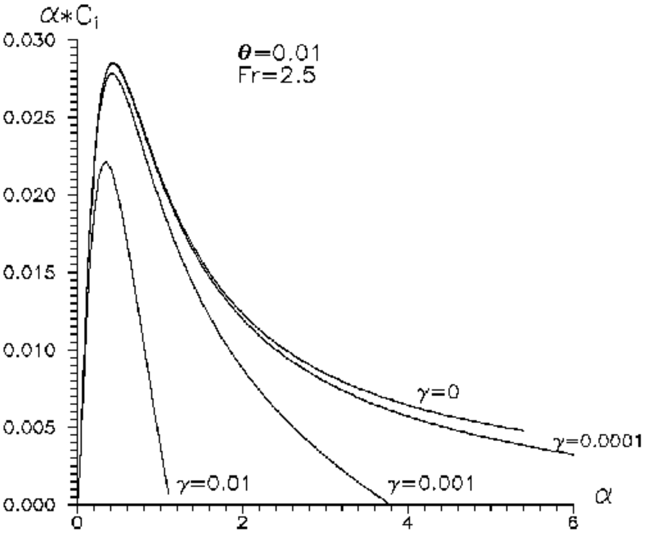
$$u(x,y,t) = u_0 + u_1(x,y,t),\ ^3 = {^3}_1(x,y,t),\ z_b = (z_b)_1(x,y,t),$$
$$h = h_0 + h_1(x,y,t),\ s = s_0 + s_1(x,y,t)$$
(4.4)

where - u_1, $^3{}_1$, s_1, $(z_b)_1$, h_1 are small indignations. The linear analysis of instability of current was carried out on the basis of the linear theory of instability. The problem was reduced to a spectral problem . Critical number Froude $Fr_c = 2$ for the superficial waves, influencing on stability of movement is determined.

5. Results of the linear analysis

Numerical calculations were carried out on various values of free parameters $^-, Fr, p_n, {^\circ}_p, d, {^3}_n, w_p, \sim_b, ¥$. Numerical calculations have shown, that the spectral problem has two unstable solutions. The first is received from parametrical continuation of the solution existing for current of a layer of a liquid above of not deformable bottom; the basic influences on its factor of amplification render two parameters - number of Froude Fr and $^-$ a corner of an inclination (instead of, one of these parameters it is possible to use factor of resistance ^{a} as they are connected by a ratio (4.2)). For this solution which we shall name superficial, critical number Froude is close to its value for current above not deformablebottom $Fr_c = 2$. Instability of the second solution which can be named ground, essentially depends on mass transfer. Results shows coordination with experimental data [1] and nearest to results [2].



Dependence of factor of amplification from wave number ~ at the fixed number of Frude for various ¥

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