MATHEMATICAL THEORY OF STEADY HEAT TRANSFER IN A THIN FILM FLOW OF A MICROPOLAR FLUID OVER AN INCLINED PERMEABLE BED

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ABSTRACT
The flow and heat transfer of a micropolar fluid over an inclined permeable bed is investigated. The flow in the thin film region is governed by micropolar fluid model whereas the flow in the permeable bed is described by Darcy law. The expressions for the fluid velocity and the microrotation are obtained. When the Darcy number tends to zero, the results reduce to the corresponding ones of Sajid et al. (2009) for the thin film flow of a micropolar fluid over an inclined rigid plane. The effect of permeability on the fluid velocity is discussed. It is observed that the magnitude of microrotation and the fluid velocity decreases with increasing micropolar parameter \( K \).

Keywords: Thin film flow, Micropolar fluid, Permeable bed, Heat transfer.

1. INTRODUCTION
The research on micropolar fluids has been of great interest because Navier–Stokes equations for Newtonian fluids cannot successfully describe the characteristics of fluid with suspended particles. Fabula [1] have shown experimentally that the fluids containing minute polymeric additives can reduce skin friction 25-30\%. This reduction was explained with the theory of micropolar fluids. Eringen [2] first developed the theory of micropolar fluids and thermomicropolar fluids to describe the characteristics of liquid crystal, polymeric fluids, and fluids containing small additives by considering the microscopic effects due to the local structure and micromotions of the fluid elements. This theory is based on the assumption that the motion of fluid micro elements is very small and takes into account the effects of microrotational surface and body couples. Excellent reviews about the applications of micropolar fluids have been written by Ariman et al. [3]. To understand the departure from the viscous fluid flow model, several problems that were studied in viscous fluid flow theory have also been studied in the realm of micropolar fluids. Gorla et al. [4] has discussed magnetohydrodynamic free convection boundary layer flow of a thermomicrofluid over a vertical plate. Lukaszewicz [5] gave many important aspects of the theory and applications of micropolar fluids.


Zueco et al. [14] analyzed the problem of unsteady MHD free convection of a micropolar fluid between two parallel porous vertical walls with convection from the ambient surroundings. Sajid et al. [15] have analyzed...
the boundary layer flow of a micropolar fluid through a porous channel using homotopy analysis method (HAM). The thin film flow of a micropolar fluid over an inclined plane is studied by Sajid et al. [16]. Umavathi [17] has studied the problem of Mixed convection of micropolar fluid in a vertical double-passage channel. The Mixed convection flow of a micropolar fluid with concentration in a vertical channel in the presence of heat source or sink is studied by Umavathi et al. [18].

In this paper, we discuss the problem of thin film flow of a micropolar fluid over an inclined permeable bed with heat transfer. The expressions for the fluid velocity, the microrotation and the temperature are determined. The results are discussed through graphs.

2. MATHEMATICAL FORMULATION OF THE PROBLEM

Consider the thin film flow of a micropolar fluid down an inclined permeable bed with permeability $k'$ (see Fig.1). Let the inclination of the bed to the horizontal be $\gamma$. The ambient air is assumed to be stationary so that the flow is caused by gravity only. Also the surface tension is assumed negligible and the thickness of the film is $\delta$. The pressure is assumed constant so that the pressure gradient becomes zero. The body force and the body couple are negligible. The flow over the permeable bed is described by micropolar fluid model whereas the flow in the permeable bed is described by Darcy law.

Let the $x$-axis (the flow direction) be taken along the permeable bed and $y$-axis be taken perpendicular to the bed. The fluid velocity $\mathbf{V}$ and micro-rotation $\mathbf{N}$ are assumed to be in the form $\mathbf{V}=[u(y),0,0]$, $\mathbf{N}=[0,0,N(y)]$.

![Fig.1: Physical Model](image)

In view of the above assumptions, the basic equations reduce to

\[
\frac{\partial u}{\partial x} = 0 \quad (1)
\]

\[
\left( v + \frac{k}{\rho} \right) \frac{d^2 u}{dy^2} + \frac{k}{\rho} \frac{dN}{dy} + g_1 \sin \gamma = 0 \quad (2)
\]

\[
\left( v + \frac{k}{2\rho} \right) \frac{d^2 N}{dy^2} - \frac{k}{\rho} j \left( 2N + \frac{du}{dy} \right) = 0 \quad (3)
\]

\[
K \frac{d^2 T}{dy^2} + \mu \left( \frac{\partial u}{\partial y} \right)^2 = 0 \quad (4)
\]

where $g_1 = -g$
where $u$ is the velocity component in $x$-direction, $N$ is the microrotation in $z$-direction, $\rho$ is the density of the fluid, $j$ is the micro inertia per unit mass, $\nu$ is the kinematic viscosity, $k$ is the vortex viscosity, $g_1$ is the acceleration due to gravity.

The Darcy velocity is

$$Q = \rho g_1 \sin \gamma$$  \hspace{1cm} (5)

The micro-rotation $N$ at the wall is related to the shear stress at the wall by the relation

$$N_w = -n \tau_w$$  \hspace{1cm} (6)

where $N_w, \tau_w$ are micro-rotation and shear stress at the wall respectively and $n$ is a constant, $0 \leq n \leq 1$.

The boundary conditions are

$$u = \frac{\sqrt{k'}}{\alpha} \frac{du}{dy} \text{ at } y = 0$$  \hspace{1cm} (7)

$$N = -n \frac{du}{dy} \text{ at } y = 0$$  \hspace{1cm} (8)

$$\frac{du}{dy} = 0 \text{ at } y = \delta$$  \hspace{1cm} (9)

$$N = 0 \text{ at } y = \delta$$  \hspace{1cm} (10)

$$T = T_1 \text{ at } y = 0$$  \hspace{1cm} (11)

$$T = T_0 \text{ at } y = \delta$$  \hspace{1cm} (12)

where $\alpha = \text{slip parameter}$, $k_1 = \text{permeability of the inclined bed}$.

### 3. NON-DIMENSIONALISATION OF THE FLOW QUANTITIES

It is convenient to introduce the non-dimensional quantities

$$y^* = \frac{y}{\delta}, \quad u^* = \frac{\delta}{\nu} u, \quad N^* = \frac{\delta^2}{\nu} N, \quad K = \frac{k}{\mu}, \quad Da = \frac{k'}{\delta^2}, \quad \theta = \frac{T - T_0}{T_1 - T_0},$$  \hspace{1cm} (13)

$$m_1 = \frac{\delta^3 g_1}{\nu^2}, \quad \text{where} \quad j = \delta$$

In view of the above non-dimensional quantities, the basic equations (1) – (4) and the boundary conditions (7) – (12) can be expressed in non-dimensional form, dropping asterisks, as

$$\frac{\partial u}{\partial x} = 0$$  \hspace{1cm} (14)

$$(1 + K) u'' + K N' + m_1 \sin \gamma = 0$$  \hspace{1cm} (15)

$$\left(1 + \frac{K}{2}\right) N'' - K \left(2N + u'\right) = 0$$  \hspace{1cm} (16)

$$\frac{d^2 \theta}{dy^2} + A \left(\frac{\partial u}{\partial y}\right)^2 = 0$$  \hspace{1cm} (17)

$$u = \beta \frac{du}{dy} \text{ at } y = 0$$  \hspace{1cm} (18)
\[
N = -n \frac{du}{dy} \quad \text{at} \quad y = 0 \quad (19)
\]
\[
\frac{du}{dy} = 0, \quad N = 0 \quad \text{at} \quad y = 1 \quad (20)
\]
\[
\theta = 1 \quad \text{at} \quad y = 0 \quad (21)
\]
\[
\theta = 0 \quad \text{at} \quad y = 1 \quad (22)
\]

where \( \beta = \frac{\sqrt{Da}}{\alpha} \), \( A = Ec Pr \)

4. SOLUTION OF THE PROBLEM

On integrating equation (15) and using the condition (20), we get
\[
(1 + K)u' + KN + m_i(y - 1) \sin \gamma = 0 \quad (23)
\]

Eliminating \( u' \) from equations (16) and (23), we get
\[
\left(1 + \frac{K}{2}\right) N'' - \frac{K(K + 2)}{K + 1} N + \frac{m_i K}{K + 1} (y - 1) \sin \gamma = 0 \quad (24)
\]

At the permeable wall \( y = 0 \), (24) can be written as
\[
(1 + K)u'(0) + K N(0) - m_i \sin \gamma = 0 \quad (25)
\]

Using (19) in (25), we get
\[
N(0) = \frac{m_i n \sin \gamma}{K(n-1)-1} \quad (26)
\]

Solving (15) subject to the conditions (26) and (21), we obtain the micro-rotation as
\[
N(y) = c_1 \left(\cosh cy - \coth c \sinh cy\right) \sin \gamma + c_2 (y - 1) \sin \gamma \quad (27)
\]

Substituting the above expression for \( N(y) \) in (23) and using (18), we obtain the fluid velocity as
\[
u(y) = c_3 \left(\sinh cy + \coth c\right) \sin \gamma + c_4 \cosh c \sin \gamma - c_5 \left(\frac{y^2}{2} - y\right) \sin \gamma + c_6 \quad (28)
\]

Substituting the above expression for \( u(y) \) in (17) and using (21) and (22) we obtain the fluid temperature as
\[
\theta(y) = -A \sin^2 \gamma \left[ a_1 \left(\frac{\cosh 2cy}{8c^2} + \frac{y^2}{4}\right) + a_2 (1 - y)^4 + a_3 \left(\frac{(1-y) \cosh cy}{c^2} + \frac{2 \sinh cy}{c^3}\right)\right] + a_4 y + a_5 \quad (29)
\]

where
\[
c = \sqrt{\frac{2K}{K+1}}, \quad c_1 = \left(\frac{m_i}{K+2} + \frac{m_i n}{K(n-1)-1}\right), \quad c_2 = \frac{m_i}{K+2},
\]
\[
c_3 = -\frac{1}{K+1} \sqrt{\frac{K(K+1)}{2}} \left(\frac{m_i}{K+2} + \frac{m_i}{K(n-1)-1}\right), \quad c_4 = \frac{1}{K+1} \sqrt{\frac{K(K+1)}{2}}
\]
\[ c_5 = \frac{K m_1}{K + 2} + m_1, \quad c_6 = \frac{1}{K + 1} \left( -\beta c_4 \frac{K m_1 \beta}{K + 2} + m_1 \beta \right) \]

\[ a_1 = A \sin^2 \gamma \left[ a_3 \frac{\cosh 2 c y}{8 c^2} + \frac{1}{4} \right] + a_5 \frac{2 \sinh c y}{c^3} \right] - a_2 \]

\[ a_2 = 1 + \sin^2 \gamma \left( \frac{a_3}{8 c^2} + a_4 + \frac{a_5}{c^2} \right) \]

\[ a_3 = \left( c c_3 \right)^2, \quad a_4 = \left( c_3 \right)^2 \frac{1}{12}, \quad a_5 = 2 c c_3 c_5 \]

We note that when Da tends to zero (27) and (28) reduce to the results of Sajid et al. (2009) for the flow of micropolar fluid over an inclined plate.

5. RESULTS AND DISCUSSION

In this paper, thin film flow of a micropolar fluid over an inclined permeable bed with heat transfer is investigated and the results are discussed for various physical parameters.

Flow solutions are depicted graphically to see the effects of micropolar fluid parameter K, concentration of microelements n, the inclination, the dimensionless parameter β, micropolar parameter m_1, on magnitudes of velocity and microrotation.

The variation of microrotation with y is calculated from equation (27) for different values of K and is shown in Fig. 2. for fixed m_1, n and γ. We observe that the microrotation decreases with increasing K. The variation of microrotation with y is calculated for different values of m_1 and γ and is shown in Figures 3 and 4 for fixed n and K. It is seen that the microrotation increases with increasing m_1 or γ.

The variation of velocity with y is calculated from equation (28) for different values of K and is shown in Fig. 5. for fixed m_1, n, β and γ. We observe that the velocity increases with increasing K. The variation of velocity with y is calculated for different values of m_1, γ and β and is shown in Figures 6, 7 and 8 for fixed n, and K. It is seen that the velocity decreases with increasing m_1 or γ or β.

From the equation (29), we have calculated the temperature as a function of y, for fixed n, β and A and for different values of m_1, γ and K and is shown in Figures 9, 10 and 11. We observe that the temperature increases with the increase in m_1 or γ or K.

![Fig.2: Microrotation distribution for various values of K for fixed m_1=1, n=1/2 and γ = π / 4.](image1.png)

![Fig.3: Microrotation distribution for various values of m_1 for fixed K=1, n=1/2 and γ = π / 4.](image2.png)
Fig. 4: Microrotation distribution for various values of $\gamma$ for fixed $m_1=1$, $n=1/2$ and $K=1$.

Fig. 5: Velocity distribution for various values of $K$ for fixed $m_1=1$, $n=1/2$, $\beta=0.4$ and $\gamma = \pi / 4$.

Fig. 6: Velocity distribution for various values of $m_1$ for fixed $K=1$, $n=1/2$, $\beta=0.4$ and $\gamma = \pi / 2$.

Fig. 7: Velocity distribution for various values of $\gamma$ for fixed $m_1=0.5$, $K=1$, $n=1/2$, and $\beta=0.4$.

Fig. 8: Velocity distribution for various values of $\beta$ for fixed $m_1=0.5$, $K=1$, $n=1/2$, and $\gamma = \pi / 4$.

Fig. 9: Temperature distribution for various values of $m_1$ for fixed $n=1/2$, $\beta=1$, $\gamma = \pi / 4$ and $A=1.65$. 
Fig.10: Temperature distribution for various values of $\gamma$ for fixed $m_1=4$, $n=1/2$, $\beta=1$, $K=1$ and $A=1.65$.

Fig.11: Temperature distribution for various values of $K$ for fixed $m_1=4$, $n=1/2$, $\beta=1$, $\gamma = \pi / 4$ and $A=1.65$.

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