MDO Framework for Conceptual Design of Closed Wing UAV

Plamen Roglev*
*Doctoral student, Department of Aviation and Transport Technology, Technical University–Sofia, Branch Plovdiv

ABSTRACT
The nonplanar wing systems that form closed loops include box-wings, ring wings and joined wings. The box-wing aircraft configuration is offering substantial aerodynamic, aeroelastic and structural advantages. It is characterized by very strong aerostructural coupling and the application of multidisciplinary design optimization (MDO) is the only way to realize its potential advantages. A methodology for MDO in the conceptual design stage that is aimed at rapid design space exploration and visualization is presented.

Keywords – closed wing, box-wing, joined wing, multidisciplinary design optimization

1. Introduction
Today mathematical modelling and optimization is a fundamental method in engineering. As the aircraft design is multidisciplinary by nature, the need to apply multidisciplinary design optimization (MDO) methods is well understood and accepted. Many MDO architectures are developed. A good summary of them can be found in [1]. Despite the advances, the complex engineered systems design remains a challenge, and many projects suffer from cost overruns and delays.

Despite the advances in computational power and software the implementation of MDO still faces major challenges:

Computational cost – problems are getting more complex and/or the designers are trying to do more. Some advanced analysis codes like CFD have very high computational cost.

Numerical noise - it is a result of incomplete convergence of iterative processes, round-off errors or the discrete representation of continuous physical phenomena. The presence of numerical noise in realistic engineering simulations often makes ineffective the use of gradient-based optimization methods. And so the computational cost is further increased when they are directly used for automated optimization.

Human interface - MDO should support human decision-making instead of trying to be an automated design tool. It should not only shorten the design time but also help engineers gain insight into the problem. As Martins points in [2], "We need effective strategies to put designers “back in the loop” (e.g., to explore design spaces, filter out ineffective system architecture, manage design freedom, handle uncertainties”.

Consequently, MDO should be aimed at augmenting and enhancing designers’ capabilities to make better informed decisions. In order to be effective, the MDO should be tailored for the job. The need to utilize specialized MDO methodology is proven by the popular “no free lunch” theorem of Wolpert, and Macready [3]. The theorem states that any two optimization algorithms are equivalent when their performance is averaged across all possible problems. Which means that if we try to use a universal optimization method for all tasks it will not be more effective than a random search. If the MDO methodology is to be effective, it should make use of the a priory available information about the design problem and be designed accordingly.

During the conceptual design phase of new aircraft designers have to evaluate a large number of different concepts, searching for the one that meets the requirements in the best way. This means that they need to iteratively cycle through sketching a concept, analyzing it and evaluating and comparing its performances. The conceptual design tools need to provide rapid exploration of the whole design space and visualization of the dependencies.

Conceptual design is usually performed by small teams and the designs produced during concept work can usually be characterized by relatively few quantities. The degree of uncertainty is high. It is present in the models, the interdisciplinary interfaces, the requirements, and the operating conditions. So uncertainty has to be confronted by applying mathematical techniques to quantify it and optimize the systems in its presence.

When selecting a MDO methodology the designers are confronted with a wide range of options. The optimization problem formulation and the solution strategy employed strongly influence the time and resources needed to find a design solution. Having outlined the major problems of MDO implementation, we consider that in order to be effective, a conceptual MDO methodology should provide rapid exploration of the design space and
provide good visualization of the dependencies of the design object, even at the expense of some accuracy. At the same time it should keep the designer informed about the degree of uncertainty present in it.

In the following parts of this article we present a MDO methodology for conceptual design of box-wing UAV that satisfies these requirements. First we will outline the peculiar characteristics of this type of aircraft.

2. Box-wing aircraft characteristics

The box-wing is a type of nonplanar wing systems that form closed loops. Closed wing configurations include also ring wings and joined wings. The box-wing aircraft configuration is promising to provide aerodynamic, aeroelastic and structural advantages over the conventional cantilever wing aircraft. The configuration is characterized by tandem wings joined at their tips by vertical planes. (Fig. 1) The joined-wing configuration proposed by Wolkovitch can be viewed as a variant of box-wing with zero height of the vertical planes.

The box-wing is characterized by very strong aerostructural coupling and the application of MDO is the only way to realize its potential advantages.

![Fig. 1 Box-wing UAV](image)

2.1 Box-wing aircraft geometric parameters

The geometric parameters that are specific to box-wing aircraft are[4]:

- $S = S_f + S_r$ - reference wing surface (sum of the reference surfaces of the front and rear wings);
- $B = \frac{b_r}{b_f}$ - ratio of span of the rear wing to the span of the front wing;
- $\overline{S} = \frac{S_r}{S_f}$ - ratio of the reference surface of the rear wing to the reference surface of the front wing;
- $\lambda_{eq} = \frac{b_{main}^2}{S}$ - aspect ratio ($b_{main} = \max(b_f, b_r)$)

$$D = \frac{d}{b}$$ Specific stagger is the ratio of the horizontal distance between the roots of both wings to the span. When the front wing is positioned above the rear wing the stagger is positive.

$$H = \frac{h}{b}$$ Specific height or gap is the ratio of the vertical distance between the roots of the wings to the span and is the parameter that influences most the induced drag of the configuration.

- $\psi_0$, $\phi_0$ - Decalage – the difference between the wing root incidence of the front and rear wings.
- The geometrical characteristics that complete the description of the wings are their respective sweep ($\chi$, $\chi_r$), dihedral($\psi$, $\psi_r$), and washout.

2.2 Box-wing aircraft stability and control features

The CG (centre of gravity) envelope of the box wing aircraft (the possible positions of the CG along the longitudinal axis) depends on the geometric and aerodynamic parameters of the front and rear wings of the aircraft, since there is no horizontal tail plane. In this way it is not possible to utilize the classical approach of specifying the CG position as a percentage of the MAC (mean aerodynamic chord). The most forward CG position is limited by controllability requirements, whilst the most aft position is limited by stability requirements. The initial conditions for static longitudinal stability are:

- **longitudinal static stability condition** - the slope of the pitching moment about the CG is negative:

$$Cm_\alpha = \frac{\partial Cm}{\partial \alpha} < 0. \quad (1)$$

- **Trim condition** - the pitching moment about the CG is positive at zero lift:

$$Cm_{\text{cl.}0} > 0 \quad (2)$$

Pitch damping

$$Cm_q = \frac{\partial Cm}{\partial (qc / 2V)} \quad (3)$$

Elevator control power

$$Cm_{\delta e} = \frac{\partial Cm}{\partial \delta e} \quad (4)$$

where $Cm$ is the pitching moment coefficient, $\alpha$ is the angle of attack, $q$ is the pitch rate, $c$ is the mean aerodynamic chord, $V$ is the flight velocity, and $\delta e$ is the elevator deflection angle.

3. MDO architecture

MDO architectures provide a structure of information flow between analysis disciplines and numerical
optimizers of an optimization problem formulation. The discipline analysis activities are usually split into areas of expertise, such as an aerodynamics group and a structures group. In a smaller scale design, an analogous split may be made into discipline-related analysis codes. The purpose of MDO architecture is to coordinate these separate disciplines in a search for a system-level optimum in an efficient way and to provide faster solution by parallel computing. Example architectures include All-in-one, concurrent subspace optimization (CSSO), bi-level integrated system synthesis (BLISS), and collaborative optimization (CO). The coupling breadth and strength of the optimization problem govern the choice of a suitable architecture. For the case of the conceptual design of a box-wing aircraft All-in-one MDO architecture should be used because of the high degree of the coupling. [6] This architecture ensures fastest convergence. A block diagram of the proposed methodology is shown on Fig.2

Another positive feature of the application of metamodels is that it decouples the MDO routine from the analysis codes. Different simulation software with different fidelity levels can be used. There exist several meta-model building methods like polynomial regression, kriging, regression splines, radial basis functions (RBF) and Artificial Neural Networks (ANN).

The two most used model classes in the analyses are the polynomial models and the kriging model. It should be noted that kriging and RBF models are interpolating models, i.e. they are exact in the given data points, whereas polynomial models and ANN are approximating models. The application of different types of metamodels is reviewed in [7]. Kriging, as the method that provides highest fidelity, is being used in many applications. But it is accompanied by higher computational complexity and has issues with numerical noise.

Given the level of uncertainty in early design phases, the results of surrogate modelling and optimization are highly uncertain in early stages even with perfect modelling fidelity. So we choose to make use of polynomial metamodels. The coefficients of the polynomial regression model are determined according to a least-squares procedure. Polynomial regression models provide global representation of the data, i.e. one regression function is used for the whole domain. Polynomial functions of different orders can be used. Most often a second order polynomial is used. Higher order models can also be built. But with higher orders polynomials there exists risk of so called over-fitting of the data. The second order polynomial regression models take the following form:

\[ y^* = \beta_0 + \sum_{j=1}^{N} \beta_j x_j + \sum_{j=1}^{N} \beta_j x_j^2 + \sum_{i=1}^{N} \sum_{j=1}^{N} \beta_{ij} x_i x_j \]  

Where N is the number of design variables, and \( \beta_0; \beta_i; \beta_{ij} \) are the regression coefficients determined by the linear regression (the model \( y^* \) is linear in the coefficients) of the polynomial model. The minimum number of points necessary is \( (N+1)(N+2)/2 \). This means that the cost of computation goes up quadratically with the number of dimensions of optimization. On Fig. 3 and 4 are given examples of second and third order polynomial metamodel of the distribution of the coefficient of lift \( C_l \) along the span of the wing in accordance to the angle of sweep of the wing.
3.2 Aerodynamic metamodelling
The most widely used method for aerodynamic analysis during the conceptual design phase is the vortex-lattice method (VLM) because of its low computational cost. However its fidelity is also low, because the VLM does not account for the boundary layer and viscosity of the fluid. The decoupling of the analysis from the optimization in the MDO methodology allows the analysis to be improved with high-fidelity CFD codes and even with physical experiment results. Panayotov and Zafirov in [8] present a method than uses RANS based correction response surface that is used to enhance the VLM analysis. This method adds little computational cost while adding substantially to the fidelity of the model. On Fig. 5 and 6 examples of some aerodynamic characteristics of a box-wing aircraft and the respective polynomial regressions are presented.

3.3 Stability and control metamodelling
The responses of the key static and dynamic longitudinal characteristics (see p. 2.2) as a function of aircraft geometry design variables are used for the construction of metamodels that serve as constraints in the optimization procedure, in order to ensure the configuration will posses the necessary stability and control features.

3.4 Structural and weight metamodelling
Structural design is usually conservative, governed by regulations and design practices. So the structural modelling should follow accepted design policies. They usually are not that complex and are easily automated. As an input for the structural model serve the aerodynamic characteristics in the form of metamodels of the distribution of the coefficient of lift force over the wing. (Fig. 7) FEM analysis of a simplified structural model is utilized for the determination of the stresses and deformations of the wings and used as an input for a sizing procedure. (Fig. 8) With it the metamodel of the structural weight response to the design parameters is created.
3.5 Uncertainty integration

During the design of complex engineered systems, uncertainty is present in the models, the interdisciplinary interfaces, the requirements, and the operating conditions. The overall goal of uncertainty modelling is to produce robust design – that is systems that perform as intended over their life cycles in the face of uncertainty. Design optimization with uncertainty involves three steps: first - identification and representation of uncertainties so as to translate the available data into mathematical models based on probability theory or nonprobabilistic approaches, second - propagating uncertainties through the numerical models to quantify their impact on the performance, and third - formulation and solution of an optimization problem with appropriate objective and constraint functions that ensure the optimum solution obtained is robust against uncertainties. Hence, at a given possibility level (also referred to as an a-cut), $\xi$ is an interval variable. In this way fuzzy modelling can be seen as a generalization of interval analysis. Results of fuzzy arithmetic can be easily interpreted as showing the possible solutions with respect to the uncertainty of inputs, while the results of stochastic simulation do not have such explicit interpretation due to the random character of the calculation.

In the proposed MDO methodology the elements of $\xi$ are modelled by fuzzy sets with appropriate membership functions. The uncertainty propagation involves computing the membership functions of the outputs of interest. This involves solving a sequence of optimization problems of the following form for various a-cuts of the input membership function:

$$f^{-}(\xi) = \min_{\xi} \in E f(\xi, \xi)$$  \hspace{1cm} (6)$$

$$f^{+}(\xi) = \max_{\xi} \in E f(\xi, \xi)$$  \hspace{1cm} (7)$$

The interval representation is the simplest nonprobabilistic approach. The uncertain parameter $\xi$ is represented by the interval $[\xi_{l} \, \xi_{u}]$ where $\xi_{l}$ and $\xi_{u}$ denote the lower and upper bound, respectively. The interval bounds are then propagated through the analysis models using interval algebra to arrive at bounds on the output variables. Usually this leads to bounds that are conservatively wide. Fuzzy set theory also provides an approach for modelling parameter uncertainty based on inexact knowledge. In standard set theory, an element either lies inside a set or outside it – according to fuzzy set theory, this is a crisp set since the degree of membership of a point is 1 if it is inside the set or 0 if outside the set. In fuzzy modelling, uncertainty is represented using sets with fuzzy boundaries. A membership function is associated with a fuzzy set, which indicates the degree of membership of a given point. The degree of membership can vary from 0 to 1 and is an interval variable. In this way fuzzy modelling can be seen as a generalization of interval analysis. Results of fuzzy arithmetic can be easily interpreted as showing the possible solutions with respect to the uncertainty of inputs, while the results of stochastic simulation do not have such explicit interpretation due to the random character of the calculation.

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This is because, for fixed $\alpha$ (possibility level), the elements of $\xi$ are interval variables. In this way we estimate its upper and lower bounds using optimization techniques. These bounds represent the most-favourable and least-favourable values of the outputs. Solving the preceding optimization problems repetitively for different values of $x$ to calculate the bounding envelope of $f(x, \xi)$ is computationally expensive. Since we use metamodels, the computational cost is reduced.

3.5 Design objectives and constraints

Of great importance is the right selection of the design objectives. They should represent not some qualities of the aircraft but the chief utility characteristics. For UAV the direct operating costs are difficult to formulate. The most widely advertised characteristic is usually the endurance with a specified payload. The cost of procurement is the most important financial characteristic. So we consider these to be the most adequate design objectives. Rather than optimizing for a single design objective a Pareto front demonstrating the trade-off between the chief performance estimator and cost. Maximum take-off weight (MTOW) can be used either as a design constraint or as an objective. Other constraints include stability and control estimates.

4. Conclusion

An effective MDO methodology for conceptual design of box-wing UAV that provides rapid exploration and visualization of the design space is presented. It allows comprehensive evaluation of this advanced aircraft configuration, which possesses many promising applications, as can be seen in [9].

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